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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

AMPLITUDE SHADING AND PHASE WEIGHTING OF A  
VERTICAL LINEAR ARRAY IN THE SOFAR CHANNEL  
BY THE LINEAR MINIMUM VARIANCE  
ESTIMATION TECHNIQUE

by

Daniel Patrick McVicar

December 1983

Thesis Advisor:

P. H. Moose

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Amplitude Shading and Phase Weighting of a Vertical Linear Array  
in the SOFAR Channel  
by the Linear Minimum Variance Estimation Technique

by

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Captain, Canadian Armed Forces  
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Submitted in partial fulfillment of the  
requirements for the degrees of  
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December 1983





# ABSTRACT

A single linear vertical passive array is used in the 'SOFAR' channel to determine the depth of a single underwater source at a constant range. The phase and amplitude weights applied to the array are determined by the linear minimum variance estimation technique. The resulting beam pattern is compared to the conventional time domain beamformer. It was found that the linear minimum variance estimation technique of amplitude shading and phase weighting was significantly superior to the conventional beamformer.



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## I. INTRODUCTION

There are many solutions to the problem of predicting the location of an underwater energy source. One common solution is the use of a passive hydrophone which detects the pressure waves radiating from the source. The hydrophone sensor is assumed to be omnidirectional and therefore incapable of estimating direction. To provide directionality a series of sensors are placed in a row to form a passive linear array.

A familiar method of determining directionality is time-domain beamforming. In this principle, it is assumed that the source is far enough away so that the pressure wave appears to be a plane wave when viewed at the site of the receiving array.

Thus a set of time delays are calculated for any direction of signal arrival, which, when applied to the receiver outputs causes them to be in phase and to reinforce when summed. The resultant angular response to signals arriving from other than the nominated direction is then a function of the array geometry, relative to the signal wavelength, and any weighting factors which have been applied to the receiver outputs. The effect is to generate a main receiving beam in the desired direction, with a series of undesired subsidiary sidelobes whose magnitude can be controlled to some extent by the choice of a suitable array geometry and the use of amplitude weightings on the receiver outputs.

In order to determine location, three or more such arrays separated by a known amount may be used.

This study is concerned with a single linear vertical passive array and the determination of the depth of a single



underwater source. The analysis is based on the following assumptions:

- The underwater source is emitting continuously and at a monochromatic frequency.
- Both the source and receiving hydrophones are stationary in space causing a constant range.
- The range is sufficiently long so that the channel is filled with R-R (refracted-refracted) rays.
- There is no distortion introduced in the propagating medium so that the signals received at each sensor are identical except for constant delays.
- The source signal and noise are independent and stationary gaussian random processes.
- The speed of sound profile is triangular and symmetric with the deep sound channel axis at 1000 meters. The velocity gradient is  $-0.017$  meters/meter/sec above 1000 meters and  $+0.017$  meters/meter/sec below 1000 meters. This profile gives a speed of sound at the surface of 1500 meters/sec.
- The speed of sound profile is constant in the horizontal plane.
- Only R-R rays are considered. All other rays have sufficient loss that their effect is negligible.

As opposed to conventional time-domain beamforming, this study makes no assumption of planar wave fronts at the receiver site. Therefore the time delays applied to each receiver will not, in general, be a linear function of depth.

Since it is desired to determine whether or not there is a source present at a specific depth the result will be a





binary decision. A "1" will indicate signal source present; a "0" will indicate signal source not present. For the constant range there will be "N" test depths investigated for the signal source. The number of hydrophones in the vertical array will be "L".

For a single source at a given depth, the travel time is calculated from the depth to each hydrophone. This travel time is converted into a phase delay for each hydrophone so that after summation from all hydrophones a maximum output is achieved. This output is then passed through a squaring device, an integrator, and a threshold and flip flop device to give a "1" binary output. If the signal source is at a different depth and the same previous phase delays are used for each hydrophone then the output will be somewhat less than the previous maximum. The difference in depth required to achieve a "0" binary output is the depth resolution of the system.

The travel times for each hydrophone are calculated for each of the "N" source depths to be considered. "N" will ordinarily be much greater than "L" so that the system will be overdetermined. An overdetermined system is one in which there are more equations than unknowns. The objective then is to calculate the phase angle and amplitude weight for each hydrophone so that a determination can be made indicating the presence or absence of a signal source at a given depth.

The method used to calculate the phase and amplitude weights is the linear minimum variance estimation technique. Linear minimum variance estimators are optimum when compared with all other estimators for gaussian problems. The method is directly applicable to overdetermined systems.

The output of the summer is calculated using the linear minimum variance amplitude and phase angles assuming a source at one of the "N" source depths and no source at the



others. The calculation is repeated for each of the depths. The result, when plotted against source depth, will be referred to as the "beam pattern" of the array in this report. (Although similar, it should not be interpreted as the angular response of an array as in the conventional definition of a beam pattern. The conventional definition loses much of its utility when the wavefronts are not planar.) Ideally the beam pattern will be maximum at the desired depth and very small at all other depths so that the binary "1" decision will be made for a source at the desired depth, and a "0" decision for sources at all others. This beam pattern is compared with the depth beam pattern of the conventional time-domain beamformer mentioned above. The purpose of this thesis is to determine, as an initial investigation, whether the linear minimum variance estimation technique, when applied to a linear vertical array, is useful in depth discrimination at long ranges in a 'SOFAR' type sound channel.



## II. GENERAL THEORY

### A. RAY ACOUSTICS

The propagation of sound in an elastic medium can be described mathematically by solutions of the wave equation using the appropriate boundary and medium conditions for a particular problem. The wave equation relating the acoustic pressure 'p' to the coordinates 'x', 'y', 'z', and the time 't', may be written as

$$\frac{d^2 p}{dt^2} = c^2 \left( \frac{d^2 p}{dx^2} + \frac{d^2 p}{dy^2} + \frac{d^2 p}{dz^2} \right) \quad (2.1)$$

where 'c' is a quantity that has the general significance of sound velocity and may vary with the coordinates.

One may approximate the solution of the wave equation using ray theory: its body of results and conclusions is called ray acoustics.

Officer [Ref. 1] describes the ray solution as a complete solution to any particular propagation problem within the validity of the approximation of the Eikonal equation to the wave equation. For these approximations to be valid neither the amplitude of the wave nor the speed of sound can change appreciably in distances comparable to a wavelength.

Thus the path of a ray through a medium in which the speed of sound varies with depth can be calculated by the application of Snell's law

$$\cos\theta/c = 1/c_0 = \text{a constant for any one ray} \quad (2.2)$$





where ' $\theta$ ' is the angle of depression made with the horizontal at a depth where the speed of sound is ' $c$ ', and ' $c_0$ ' is the speed at a depth (real or extrapolated) where the ray would become horizontal.

In a medium in which the velocity of sound changes linearly with depth the sound rays can be shown to be arcs of circles, that is, to have a constant radius of curvature. Kinsler et al. [Ref. 2] give a simple and heuristic demonstration of the circularity of rays in a medium with a linear sound speed gradient ' $g$ '. The center of the circle which creates the arc lies at a depth where the sound speed extrapolates to zero. To understand this, consider a portion of a ray path with a local radius of curvature ' $R$ ', as illustrated in Figure 2.1. Since the gradient ' $g$ ' for this case is

$$g = \Delta c / \Delta z = (c_2 - c_1) / (d_2 - d_1) = (c_2 - c_1) / R (\cos \theta_1 - \cos \theta_2) \quad (2.3)$$

where ' $\Delta c$ ' is the change in sound speed and ' $\Delta z$ ' is the change in depth. It can be seen that the radius of curvature is given by

$$R = -c_0 / g = -c / (g \cos \theta) \quad (2.4)$$

The ray path is therefore a circle when ' $g$ ' is constant because ' $R$ ' is then constant. The center of curvature of a circle lies at the depth where  $\theta$  is 90 degrees, which corresponds to  $c=0$ . For the situation in Figure 2.1 the speed gradient is negative so that ' $R$ ' is positive. If the speed gradient were positive ' $R$ ' would be negative, and the path would curve upward.

Once the radius of curvature of each segment of a path is known the actual path can be traced graphically or



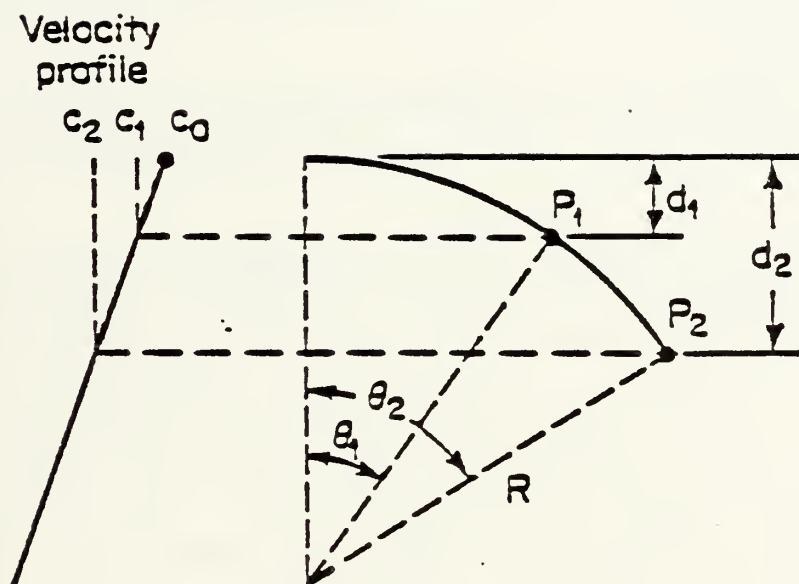


Figure 2.1 Circular Ray Path



computed. If the initial angle of depression of a ray is  $\theta_1$ , then by referring to the geometry of Figure 2.1 the changes in both range and depth are

$$\Delta r = c_1 (\sin(\theta_1) - \sin(\theta_2)) / (g \cos(\theta_1)) \quad (2.5)$$

$$\Delta z = c_1 (\cos(\theta_2) - \cos(\theta_1)) / (g \cos(\theta_1)) \quad (2.6)$$

The sign convention for these equations is: downward, to the right, and depression angles below the horizontal axis positive.

The symmetric triangular sound speed profile assumed in the introduction is similar to speed profiles encountered in the deep sound channel, sometimes called the SOFAR channel. The velocity minimum which occurs at the axis of the sound channel causes the sea to act like a kind of lens; above and below the minimum, the velocity gradient continually bends the sound rays toward the depth of minimum velocity. A portion of the power radiated by a source in the deep sound channel accordingly remains within the channel and encounters no acoustic losses by reflection from the surface and bottom. These rays are called R-R (refracted-refracted) and since they have very low transmission loss, very long ranges can be obtained from a source of moderate acoustic power output. Thus an energy source at a specific depth will propagate energy in all directions but only the direction which is toward the receiving array and for which the rays are R-R is of interest. To determine the range of depression angles which will yield R-R rays Snell's law is used to give



$$\theta_{\max} = \arccos (c_1 / (c_1 - (g d))) \quad (2.7)$$

where ' $\theta_{\max}$ ' is in radians and ' $c_1$ ' is the speed of sound at the source depth ' $d$ '.

An example is given in Figure 2.2 of a single ray trace propagating in the SOFAR channel to show how equations 2.5 and 2.6 are used in determining range and depth. The ray path is broken up into arcs of circles as shown in the figure, and then by paying close attention to the previously defined sign conventions, the change in range and depth is found. Being more specific, for arc 1,  $\theta_1$  and  $\theta_2$  are both positive; for arc 2,  $\theta_1$  is positive and  $\theta_2$  is zero; for arc 3,  $\theta_1$  is zero and  $\theta_2$  is negative; and for arc 4,  $\theta_1$  is negative and  $\theta_2$  is zero. By keeping a running total of all the depth and range changes it is possible to determine the total horizontal distance travelled and the depth at that distance.

For the speed of sound profile assumed in the introduction a computer generated ray plot is shown in Figure 2.3 for a source depth of 300 meters. As each ray propagates out from the source the triangular channel becomes filled with sound. If a receiving hydrophone is placed a great distance away, a number of refracted propagation paths will exist, each having a different travel time and crossing the channel axis at different intervals. The path with the greatest excursion from the axis will have the shortest travel time.

Officer [Ref. 1] shows that the travel time ' $t$ ' of a ray, which is an arc of a circle, is given by

$$t = \frac{1}{g} \int_{\theta_1}^{\theta_2} \frac{d\theta}{\cos \theta} \quad (2.8)$$

and the travel time for each arc is





$$t = - \frac{1}{g} \log_e \left[ \frac{\tan((\pi/4) + (\theta_2/2))}{\tan((\pi/4) + (\theta_1/2))} \right] \quad (2.9)$$

Equation 2.9, applied using the same convention as equations 2.5 and 2.6, determines the total travel time of a ray in the deep sound channel.

## B. ARRAY MODEL

The receiving linear vertical array is presumed to consist of 'L' hydrophones as shown in Figure 2.4. It is assumed that the source is emitting energy at a constant frequency, 'f', and amplitude, 'A', regardless of the depth. The source signal at the source is  $A \exp(j2\pi ft)$ . The inherent received signal at the first hydrophone is

$$x_1(t) = A \exp(j2\pi f(t-t_1)) \quad (2.10)$$

where 't<sub>1</sub>' is the travel time from the energy source to the first hydrophone and 'A' is the amplitude of the signal at the range of the array. After passage through the amplitude weight 'a<sub>1</sub>' and a phase delay of 'τ<sub>1</sub>', the signal on the first hydrophone at the input to the summer is

$$y_1(t) = A a_1 x_1(t - \tau_1) = A a_1 \exp(j2\pi f(t - t_1 - \tau_1)) \quad (2.11)$$

A time delay, for monochromatic signals corresponds to a phase shift

$$\theta = 2\pi f t \quad (2.12)$$

where 'θ' is the phase shift in radians and 't' is the time delay in seconds. Thus equation 2.11 can be written as



$$y_1(t) = A' a_1 \exp(j(2\pi ft - \phi_1 - \theta_1)) \quad (2.13)$$

where ' $\phi_1 = 2\pi ft_1$ ' is the phase delay due to the travel time from the source to the first hydrophone and ' $\theta_1$ ' is the phase delay in the receiver on the first hydrophone.

Combining all the hydrophones in the array in a summer gives as an expression for the array output

$$Y(t) = \sum_{k=1}^L A' a_k \exp(j(2\pi ft - \phi_k - \theta_k)) \quad (2.14)$$

where ' $\phi_k$ ' represents the phase delay due to the travel time from the source to the "k th" hydrophone and ' $\theta_k$ ' is the phase delay in the receiver on the "k th" hydrophone. If the amplitude of the energy source is normalized by setting  $A'=1$ , and equation 2.14 is written in terms of real and imaginary components, we have

$$Y(t) = \sum_{k=1}^L a_k \cos(-\phi_k - \theta_k) + j \sin(-\phi_k - \theta_k) \exp(j2\pi ft) \quad (2.15)$$

When an energy source is at the "q th" depth, we wish to have each receiving hydrophone's phase delay cancel out the effect of the travel time from the source to it, such that ' $-\phi_k - \theta_k$ ' is equal to a multiple of ' $2\pi$ '. This will put all signals into the summer in phase and thus maximize the signal gain for a source at the "q th" depth. From equation 2.15,

$$\sum_{k=1}^L (a_{kq} \cos(-\phi_{kq} - \theta_{kq})) = L \quad (\text{source present at } q) \quad (2.16)$$

$$\sum_{k=1}^L (a_{kq} \sin(-\phi_{kq} - \theta_{kq})) = 0 \quad (\text{source present at } q) \quad (2.17)$$



Note that the first subscript on the phase angle indicates the receiving hydrophone and the second subscript indicates the depth of the source. Thus ' $\phi_{kq}$ ' would indicate the phase shift relating to the travel time from the "q th" test depth to the "k th" hydrophone in the receiving array.

It is desirable for ' $Y(t)$ ' to be a minimum value for sources at other than the depth being investigated. Thus for each of the other ' $N-1$ ' depths ' $Y(t)$ ' is set to zero. This gives ' $N-1$ ' equations for the real terms of ' $Y(t)$ ' set to zero

$$\sum_{k=1}^L a_{kq} \cos(-\phi_{km} - \theta_{kq}) = 0 \quad (\text{source absent; } m \neq q) \quad (2.18)$$

and ' $N-1$ ' equations for the imaginary terms of ' $Y(t)$ ' set to zero

$$\sum_{k=1}^L a_{kq} \sin(-\phi_{km} - \theta_{kq}) = 0 \quad (\text{source absent; } m \neq q) \quad (2.19)$$

By using elementary trigonometric identities, equation 2.16 (real terms with source present at "q th" depth) becomes

$$\sum_{k=1}^L a_{kq} [\cos(\phi_{kq}) \cos(\theta_{kq}) - \sin(\phi_{kq}) \sin(\theta_{kq})] = L \quad (2.20)$$

Equation 2.18 (real terms with source absent for each of the other ' $N-1$ ' depths) becomes

$$\sum_{k=1}^L a_{kq} [\cos(\phi_{km}) \cos(\theta_{kq}) - \sin(\phi_{km}) \sin(\theta_{kq})] = 0 \quad m=1,2,\dots,N; m \neq q \quad (2.21)$$

Equation 2.17 (imaginary terms with source present at the "q th" depth) becomes



$$\sum_{k=1}^L -a_{kq} (\sin(\phi_{kq}) \cos(\theta_{kq}) + \cos(\phi_{kq}) \sin(\theta_{kq})) = 0 \quad (2.22)$$

and equation 2.19 (imaginary terms with source absent for each of the other 'N-1' depths) becomes

$$\sum_{k=1}^L -a_{kq} [\sin(\phi_{km}) \cos(\theta_{kq}) + \cos(\phi_{km}) \sin(\theta_{kq})] = 0 \quad m=1,2,\dots,N; m \neq q \quad (2.23)$$

Thus, there are a total of '2N' equations with '2L' unknowns.

In order to simplify, we put these '2N' equations into matrix form. Arbitrarily the real terms are made the first 'N' equations and the imaginary terms the second 'N' equations. The first real and first imaginary equation is at the lowest (shallowest) source depth and equations increase in order after that until the last real and last imaginary equation correspond to a source at the deepest depth. The resultant matrix equation becomes:

$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ L \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\phi_{11} & \cos\phi_{21} & \dots & \cos\phi_{L1} & -\sin\phi_{11} & -\sin\phi_{21} & \dots & -\sin\phi_{L1} \\ \cos\phi_{12} & \cos\phi_{22} & \dots & \cos\phi_{L2} & -\sin\phi_{12} & -\sin\phi_{22} & \dots & -\sin\phi_{L2} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \cos\phi_{1q} & \cos\phi_{2q} & \dots & \cos\phi_{Lq} & -\sin\phi_{1q} & -\sin\phi_{2q} & \dots & -\sin\phi_{Lq} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \cos\phi_{1N} & \cos\phi_{2N} & \dots & \cos\phi_{LN} & -\sin\phi_{1N} & -\sin\phi_{2N} & \dots & -\sin\phi_{LN} \\ -\sin\phi_{11} & -\sin\phi_{21} & \dots & -\sin\phi_{L1} & -\cos\phi_{11} & -\cos\phi_{21} & \dots & -\cos\phi_{L1} \\ -\sin\phi_{12} & -\sin\phi_{22} & \dots & -\sin\phi_{L2} & -\cos\phi_{12} & -\cos\phi_{22} & \dots & -\cos\phi_{L2} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ -\sin\phi_{1N} & -\sin\phi_{2N} & \dots & -\sin\phi_{LN} & -\cos\phi_{1N} & -\cos\phi_{2N} & \dots & -\cos\phi_{LN} \end{bmatrix} \begin{bmatrix} a_{1q} \cos\theta_{1q} \\ a_{2q} \cos\theta_{2q} \\ \vdots \\ a_{Lq} \cos\theta_{Lq} \\ a_{1q} \sin\theta_{1q} \\ a_{2q} \sin\theta_{2q} \\ \vdots \\ a_{Lq} \sin\theta_{Lq} \end{bmatrix} \quad (2.24)$$

Simplifying further, equation 2.24 becomes:





$$\begin{bmatrix} 0 \\ \vdots \\ L \\ \vdots \\ 0 \\ \hline 0 \end{bmatrix} = \begin{bmatrix} \underline{\alpha} & -\underline{\beta} \\ \hline -\underline{\beta} & -\underline{\alpha} \end{bmatrix} \begin{bmatrix} \underline{c} \\ \hline \underline{s} \end{bmatrix} \quad (2.25)$$

where ' $\alpha$ ', ' $\beta$ ', ' $c$ ', and ' $s$ ' represent the appropriate submatrices.

Then, by noting that the multiplication of each element of a column by the same nonzero constant doesn't affect the solution, equation 2.25 becomes:

$$\begin{bmatrix} 0 \\ \vdots \\ L \\ \vdots \\ 0 \\ \hline 0 \end{bmatrix} = \begin{bmatrix} \underline{\alpha} & \underline{\beta} \\ \hline -\underline{\beta} & \underline{\alpha} \end{bmatrix} \begin{bmatrix} \underline{c} \\ \hline \underline{s} \end{bmatrix} \quad (2.26)$$

Finally letting  $Z$ ,  $A$ , and  $\Theta$  represent the matrices in equation 2.26, we obtain

$$\underline{Z} = \underline{A}\underline{\Theta} \quad (2.27)$$

where a matrix is denoted by a capitalized underlined letter.

In summary,  $Z$  is the desired response of the vertical array to the ' $N$ ' source test depths.  $A$  represents known travel times from each of the ' $N$ ' source depths to each of the ' $L$ ' receiving hydrophones.  $\Theta$  is unknown. It is the phase and amplitude weighting which must be applied to the vertical array in order to realize  $Z$ .  $\Theta$  contains ' $2L$ ' unknowns.



Equation 2.27 represents '2N' equations. Since this system of equations is overdetermined ( $N > L$ ) an exact solution does not exist. In order to make the best estimate of  $\underline{\theta}$  for the desired response, the linear minimum variance estimation technique is used.

### C. LINEAR MINIMUM VARIANCE METHOD

Equation 2.27 represents a noise free environment. If noise were present it would become

$$\underline{Z} = \underline{A}\underline{\theta} + \underline{n} \quad (2.28)$$

with ' $\underline{n}$ ' a "2N" element column vector. This represents the noise at each source depth. ' $\underline{Z}$ ' is a linear function of ' $\underline{\theta}$ ' and is called the observation. ' $\underline{A}$ ' is a "2Nx2L" modulation or observation matrix which is known, and ' $\underline{\theta}$ ' is the "2L" element random parameter vector which is to be estimated. Assume the first and second moments of ' $\underline{\theta}$ ' and ' $\underline{n}$ ' are given by

$$E(\underline{\theta}) = \underline{\mu}_{\theta} \quad \text{Var}(\underline{\theta}) = \underline{V}_{\theta} \quad (2.29)$$

and

$$E(\underline{n}) = \underline{0} \quad \text{Var}(\underline{n}) = \underline{V}_n \quad (2.30)$$

where 'E' represents expectation or first moment and 'Var' represents the covariance matrix. It is assumed that the parameter ' $\underline{\theta}$ ' and the noise ' $\underline{n}$ ' are uncorrelated.

A restriction imposed is that the estimate must be a weighted linear combination of the observations:



$$\hat{\underline{\theta}}_L = \underline{b} + \underline{E}\underline{Z} \quad (2.31)$$

where '^' indicates estimate. The objective is to select 'b' and 'B' in order to minimize the error variance. Such an estimator is the linear minimum variance estimator; it is the best, in the sense of minimum-error-variance linear estimators.

Another restriction is that the estimator be unbiased; in other words it is required that the expected value of the estimator ' $\hat{\underline{\theta}}_L$ ' is equal to the expected value of the parameter 'θ'. Thus,

$$E(\hat{\underline{\theta}}_L) = \underline{b} + \underline{E}E(\underline{Z}) = E(\underline{\theta}) = \underline{\mu}_\theta \quad (2.32)$$

yielding

$$\underline{b} = \underline{\mu}_\theta - \underline{E}\underline{A}\underline{\mu}_\theta \quad (2.33)$$

Substituting this result in equation 2.31 gives for the unbiased linear estimator

$$\hat{\underline{\theta}}_L = \underline{\mu}_\theta + \underline{E}(\underline{Z} - \underline{A}\underline{\mu}_\theta) \quad (2.34)$$

Note that since the estimator is unbiased, the estimation error ' $\underline{\theta}_E = \underline{\theta} - \hat{\underline{\theta}}_L$ ' is zero mean. The next step is to select 'B' in order to minimize the error variance. However, this optimization problem is ill-defined because the error variance is a matrix. Therefore in order to introduce a scalar goodness measure the sum of the variances of each component of 'θ' is minimized. This is the sum of the main diagonal terms of the covariance matrix and is defined as the trace of the matrix.



$$\text{tr} (\text{Var}(\underline{\theta}_E)) = \sum_{n=1}^{2N} \text{Var}((\underline{\theta}_E)_n) \quad (2.35)$$

where 'tr' indicates trace. 'B' is then selected to minimize the trace of the error variance, or

$$\min_B \text{tr} (\text{Var}(\underline{\theta}_E)) = \min_B \text{tr} (E(\underline{\theta}_E \underline{\theta}_E^T)) \quad (2.36)$$

where '<sup>T</sup>' indicates the transpose. The following problem is then obtained by substituting equation 2.34 into equation 2.36:

$$\min_B \text{tr} (\text{Var}(\underline{\theta}_E)) = \min_B \text{tr} (E((\underline{\theta} - \underline{u}_\theta - \underline{B}(\underline{Z} - \underline{A}\underline{u}_\theta))(\underline{\theta} - \underline{u}_\theta - \underline{B}(\underline{Z} - \underline{A}\underline{u}_\theta))^T)) \quad (2.37)$$

It is well known [Ref. 3] that equation 2.37 is minimized when

$$\text{Cov}(\underline{\theta}, \underline{Z}) - \underline{B} \text{Var}(\underline{Z}) = \underline{0} \quad (2.38)$$

where 'Cov(θ, Z)' is the covariance matrix of the unknown parameters and the observations. Denoting the optimum filter by 'B<sup>\*</sup>', then if

$$\underline{B}^* = \text{Cov}(\underline{\theta}, \underline{Z}) (\text{Var}(\underline{Z}))^{-1} \quad (2.39)$$

a minimum is achieved for the sum of the squares of the errors.

Using equation 2.28 for 'Z', the covariance of 'θ' and 'Z' becomes

$$\text{Cov}(\underline{\theta}, \underline{Z}) = \text{Cov}(\underline{\theta}, \underline{A}\underline{\theta} + \underline{n}) = \underline{V}_\theta \underline{A}^T \quad (2.40)$$

since 'θ' and 'n' are uncorrelated. The variance of 'Z' is





$$\text{Var}(\underline{Z}) = \text{Var}(\underline{A}\underline{\theta} + \underline{n}) = \underline{A}\underline{V}_{\theta}\underline{A}^T + \underline{V}_n \quad (2.41)$$

Substituting these into equation 2.39 gives

$$\underline{B}^* = \underline{V}_{\theta}\underline{A}^T(\underline{A}\underline{V}_{\theta}\underline{A}^T + \underline{V}_n)^{-1} \quad (2.42)$$

and the linear minimum variance estimator is

$$\hat{\underline{\theta}}_{\text{LMV}} = \underline{\mu}_{\theta} + \underline{V}_{\theta}\underline{A}^T(\underline{A}\underline{V}_{\theta}\underline{A}^T + \underline{V}_n)^{-1}(\underline{Z} - \underline{A}\underline{\mu}_{\theta}) \quad (2.43)$$

By utilizing a matrix inversion lemma [Ref. 3] equation 2.43 becomes

$$\hat{\underline{\theta}}_{\text{LMV}} = (\underline{A}^T\underline{V}_n^{-1}\underline{A} + \underline{V}_{\theta}^{-1})^{-1}(\underline{A}^T\underline{V}_n^{-1}\underline{Z} + \underline{V}_{\theta}^{-1}\underline{\mu}_{\theta}) \quad (2.44)$$

The advantage of this equation over equation 2.43 is the size of the matrix to be inverted. In equation 2.43 the matrix has dimensionality '2N' while in equation 2.44 its dimensionality is only '2L'. Thus the advantages of the linear variance estimator are the ease with which they are derived, the mathematical tractability of the linear form, and the minimum amount of stochastic information required for development. An interesting characteristic is that the linear minimum variance estimate is the orthogonal projection of ' $\theta$ ' onto the space spanned by the observation ' $Z$ '. Because of these factors this estimator is a popular form for estimating unknowns in overdetermined equations.

For this thesis it is assumed that the noise samples are uncorrelated and identically distributed so that:

$$\underline{V}_n = \sigma^2 \underline{I}_n \quad (2.45)$$



No previous knowledge is assumed about ' $\theta$ '. This implies an infinite variance matrix which is represented as:

$$\underline{v}_{\theta}^{-1} = \underline{0} \quad \text{and} \quad \underline{\mu}_{\theta} = \underline{0} \quad (2.46)$$

The linear minimum variance estimate given by equation 2.44 is then

$$\hat{\underline{\theta}}_{\text{LMV}} = (\underline{A}^T \underline{A})^{-1} \underline{A}^T \underline{Z} \quad (2.47)$$

By determining ' $\hat{\underline{\theta}}_{\text{LMV}}$ ', the phase and amplitude weights are found for a signal source on the ' $q$  th' depth. Recall that

$$\hat{\underline{\theta}}_{\text{LMV}} = \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \\ \vdots \\ \hat{\theta}_L \\ \hat{\theta}_{L+1} \\ \vdots \\ \hat{\theta}_{2L} \end{bmatrix} = \begin{bmatrix} a_{1q} \cos \theta_{1q} \\ a_{2q} \cos \theta_{2q} \\ \vdots \\ a_{Lq} \cos \theta_{Lq} \\ a_{1q} \sin \theta_{1q} \\ \vdots \\ a_{Lq} \sin \theta_{Lq} \end{bmatrix} \quad (2.48)$$

Upon solving, this equation gives for the phase delay

$$\hat{\theta}_{mq} = \arctan(\hat{\theta}_{L+m} / \hat{\theta}_m) \quad \text{where } m=1 \text{ to } L \quad (2.49)$$

and amplitude weight

$$\hat{a}_{mq} = \hat{\theta}_m / \cos(\hat{\theta}_{mq}) \quad \text{where } m=1 \text{ to } L \quad (2.50)$$

When these amplitude weights and phase delays are applied to the vertical linear array a resulting beam pattern is formed which in the absence of noise is:



$$\hat{\underline{Z}} = \underline{A} \hat{\underline{\theta}}$$

(2.51)

The resulting beam pattern ' $\hat{\underline{Z}}$ ' can then be compared with the desired beam pattern ' $\underline{Z}$ ', as well as with a conventional beam pattern ' $\underline{Z}$ ' obtained using linear phase shifts across the array aperture selected to "steer" the array to the dominant arrival angle for the selected source depth.



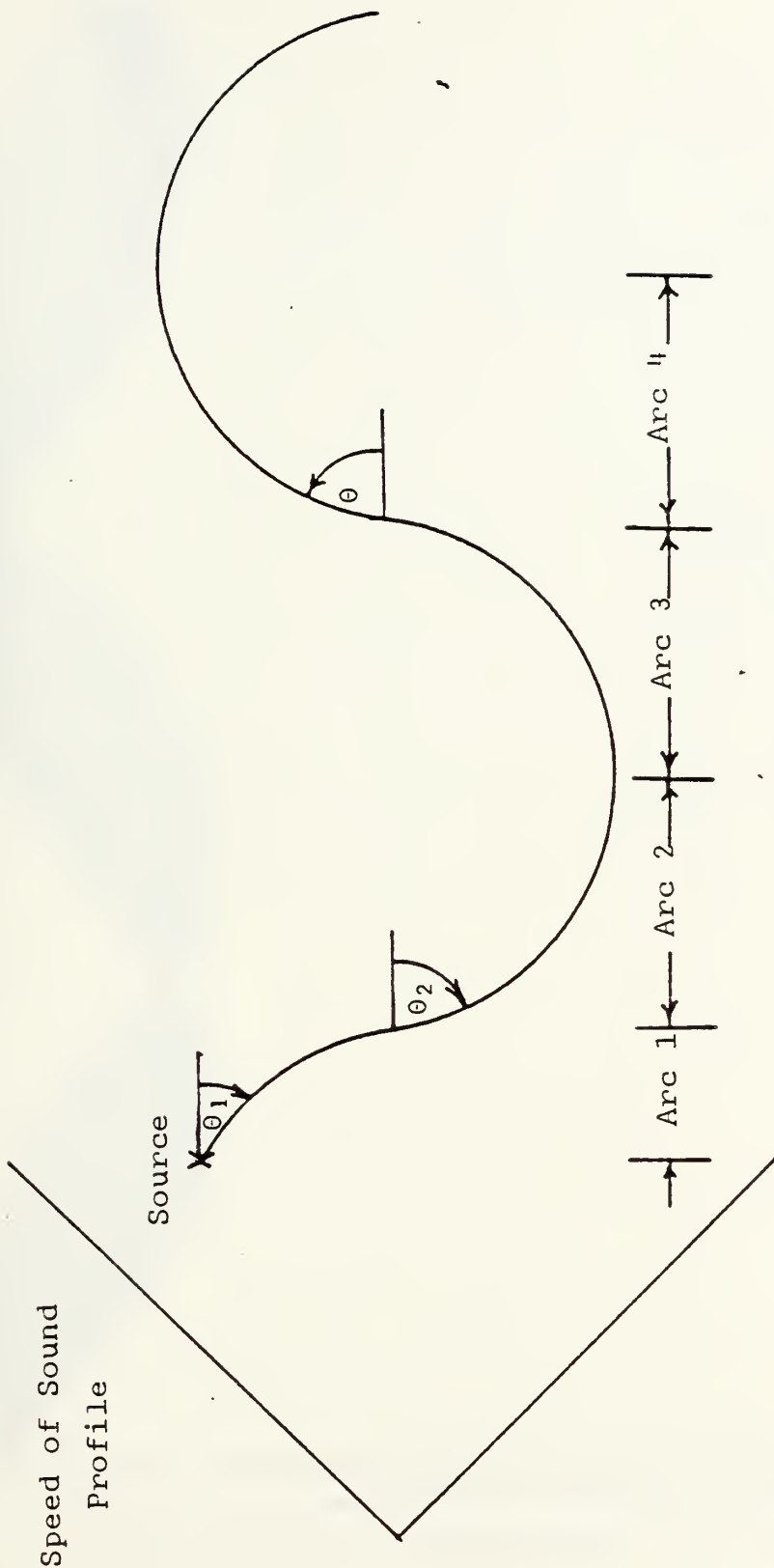


Figure 2.2 Single Ray Path Plot In Triangular SOFAR Channel





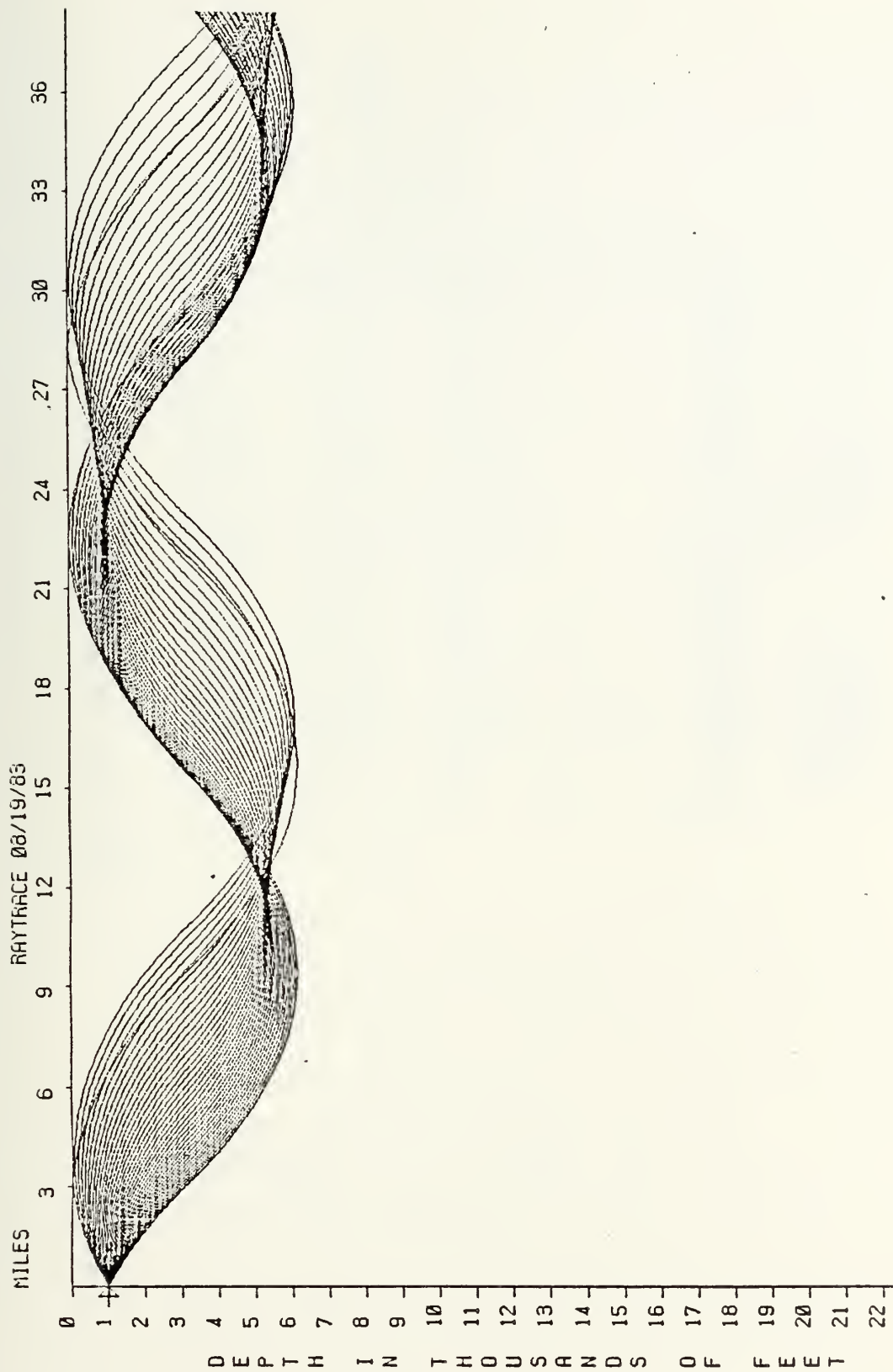


Figure 2.3 Ray Plot For Assumed Sound Channel



# Vertical Linear Array

Source Depths

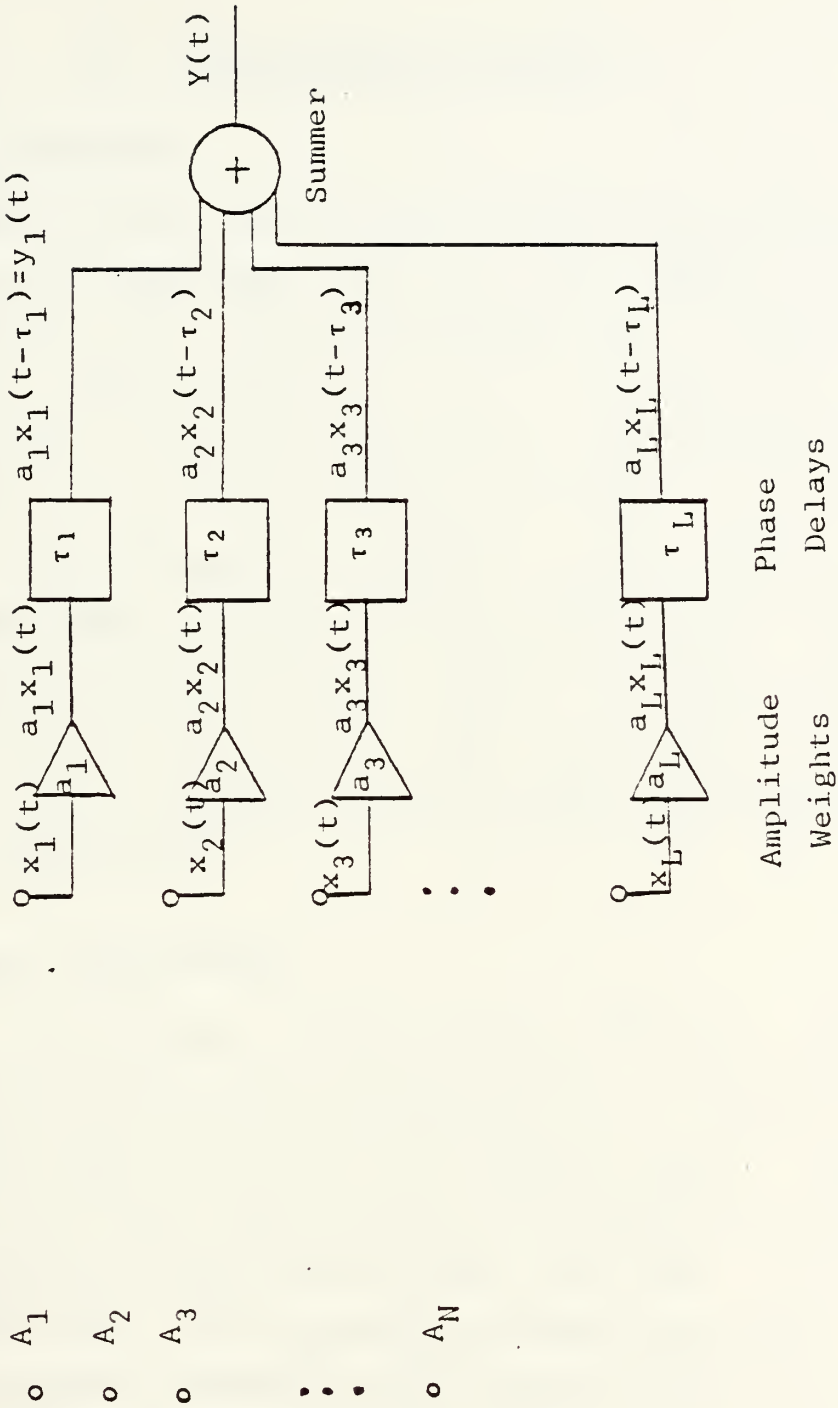


Figure 2.4 Array Model



### III. EXPERIMENTAL PROCEDURE

#### A. BASIC ASSUMPTIONS

The speed of sound profile given in the Introduction was used to test the technique, with 'L', the number of source depths chosen as 20. The source depths are at 220 meters and every 20 meters thereafter to 600 meters inclusive. 'N', the number of hydrophones used in the vertical array was chosen as 5. The hydrophones are placed at depths of 100, 200, 300, 400, and 500 meters. Thus, there are 40 equations, two for each source depth, and 10 unknowns, two for each hydrophone. A range of 200 kilometers (km) assures that the deep sound channel is filled with sound over the aperture of the vertical linear array. A frequency of 100 Hz provides good resolution in the beam pattern without introducing alias mainlobes at the selected range across the depths of investigation.

#### B. 'A' MATRIX CALCULATION

Since the gradient of the sound velocity, 'g' is constant in the area of the source depths, and the speed of sound at the surface is known, equation 2.3 is used to solve for 'c2', the speed of sound at the source depth. Equation 2.7 is used for each source depth to calculate the maximum initial depression angle which yields R-R rays. In determining travel time, only depression angles from each source which are positive (downward) and yield R-R rays are used.

Beginning with the first source depth of 220 meters, an initial depression angle of 0.0 degrees is selected. The ray path is then calculated using equations 2.5 and 2.6 and broken into a series of arcs as in figure 2.2. Equation 2.9



is used to determine the travel time for each arc in the same manner. Each arc's horizontal range, travel time, and depth are summed. When the summed horizontal range reaches 200 km the summation process ends. The travel time and depth of the ray at this horizontal range is then known. The same procedure is repeated for an initial depression angle of 0.1 degrees and every increment of 0.1 degrees thereafter until the maximum depression from equation 2.7 is reached. The final ray path is at this maximum depression angle.

The same procedure is repeated for each of the other 19 source depths.

Thus, for each initial angle from each source depth there is a ray which has a travel time and a depth when it reaches the horizontal range of 200 km. Since it is the profile of the sound pressure wave which impinges on the vertical array which is of importance, a constant can be subtracted from these calculated travel times. This constant is selected to be the travel time for the source depth of 220 meters which has an initial depression angle of 0 degrees. It is subtracted from each of the travel times making the resultant travel times relative with respect to the ray which has a 0 degree depression angle from the 220 meter depth. The program and its listing which calculates the relative travel times and the depths of these rays at the horizontal range of 200 km is given in Appendix A.

A plot of the relative travel times versus depth for the 220 meter source depth is shown in Figure 3.1. Figure 3.2 displays the plot for the 380 meter source depth. Negative relative travel times in the plots indicate that the overall travel time is less than the reference. These rays arrive at the 200 km horizontal distance before the reference ray.

Since the receiving hydrophones are at set vertical positions (100, 200, 300, 400, and 500 meters), an





interpolation is done to determine relative travel times to them from each source depth. The interpolation program and its listing is in Appendix B. Sometimes more than one R-R ray travels from the source depth to a hydrophone. When this occurs, the ray which arrives first is used in the calculation of relative travel time to that hydrophone.

Equation 2.12 determines the phase shift relating to the relative travel times. The 'A' matrix is formed by taking the appropriate sine and cosine values as in equation 2.24. The 'A' matrix is '40 by 10'.

### C. 'Z' MATRIX

Referring to equation 2.26, the 'Z' matrix is a '40 by 1' column vector. It is the desired beam pattern. The bottom 20 rows give the imaginary terms and are set to zero. The top 20 rows represent the value of the real terms at each source depth. Therefore each of the top 20 rows is set to zero except for the row containing the source. It is set to 1. For example, if the source is at 220 meters then only the top row is set to 1. If the source is at 380 meters then only the ninth row is set to 1.

### D. RESULTING BEAM PATTERN

#### 1. Using the Linear Minimum Variance Method

' $\hat{e}_{LMV}$ ' is calculated using equation 2.47. The resulting beam pattern ' $\hat{Z}$ ' is calculated using equation 2.51. The program which calculates the 'A' matrix, uses it in determining ' $\hat{\theta}_{LMV}$ ', and then calculates 'Z' is given in Appendix C. The program listing is also included.



## 2. Using Linear Phase Shifts

The conventional beam pattern is determined by using equation 2.51 where ' $\hat{\theta}$ ' is calculated by approximating the plot of relative travel time vs. depth by a straight line at the receiving hydrophone depths. For example Figure 3.3 represents this plot for the 380 meter source depth. The straight line is determined by a least squares linear regression which minimizes the sum of the squares of the deviations of the actual data points from the straight line of best fit. Note that only data points which are on the dominant curve are used in calculating the straight line. From the straight line, relative travel times to the receiving hydrophones are calculated. The relative travel times for the 380 meter source depth are given in Table I. They correspond to a plane wave arrival angle of 3.73 degrees. ' $\hat{\theta}$ ' is determined by converting these relative travel times to phase delays using equation 2.12 and then taking the appropriate sine and cosine values of these phase delays as in equation 2.24. The amplitude weights are initially assumed to be unity.

A second method for obtaining the conventional beam pattern is calculated by the same procedure except the amplitude weights which are determined by equation 2.50, the ' $\hat{\theta}_{LMV}$ ' amplitude weights, are applied to each hydrophone.



TABLE I

Relative Travel Times For 380 Meter Source Depth

<u>Hydrophone Depth</u>	<u>Relative Travel Time</u>
100 meters	-0.07565509 sec.
200 meters	-0.07131599 sec.
300 meters	-0.06697690 sec.
400 meters	-0.06263780 sec.
500 meters	-0.05829871 sec.



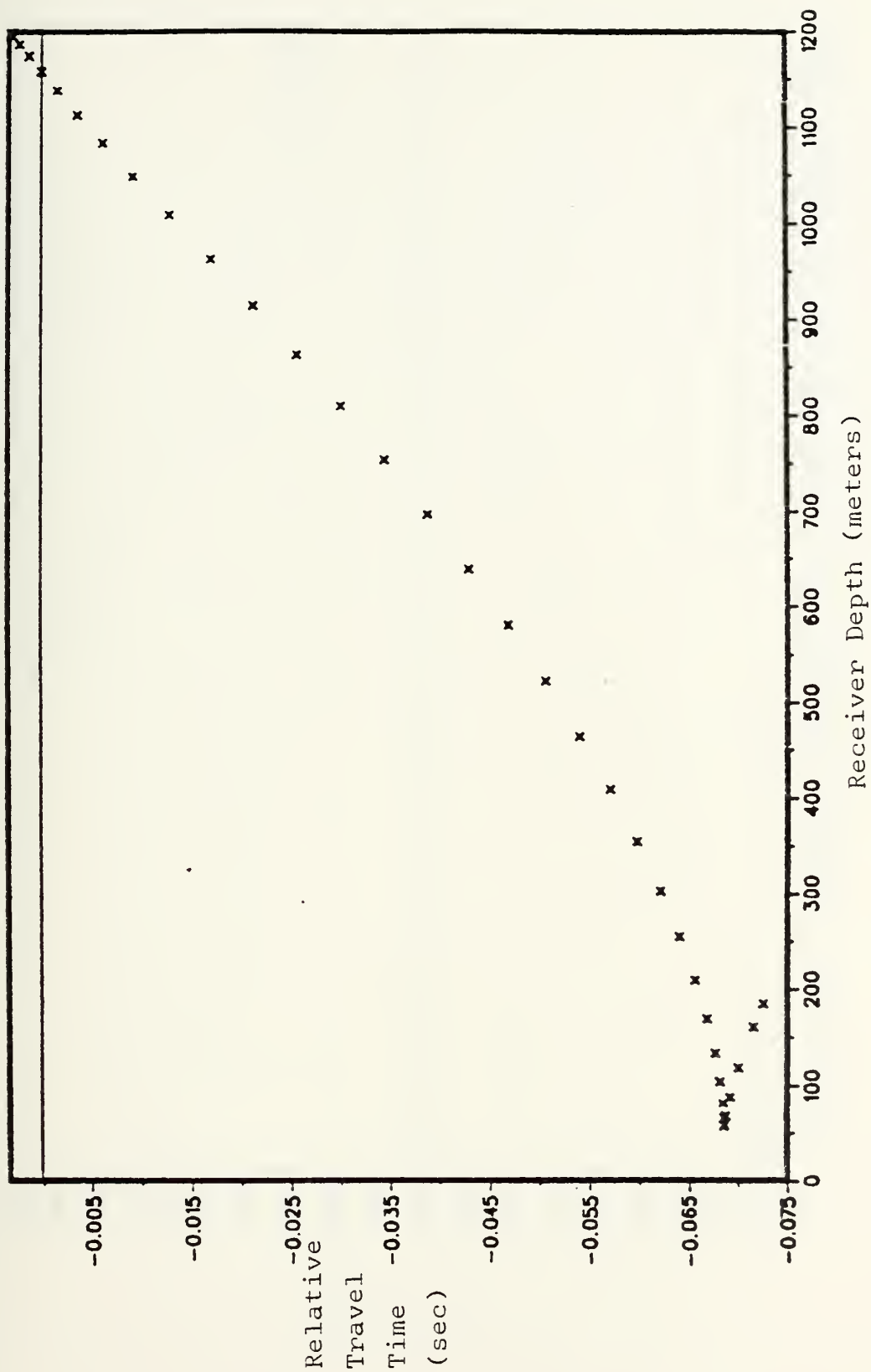


Figure 3.1 Relative Travel Time vs. Depth (220 meter source)





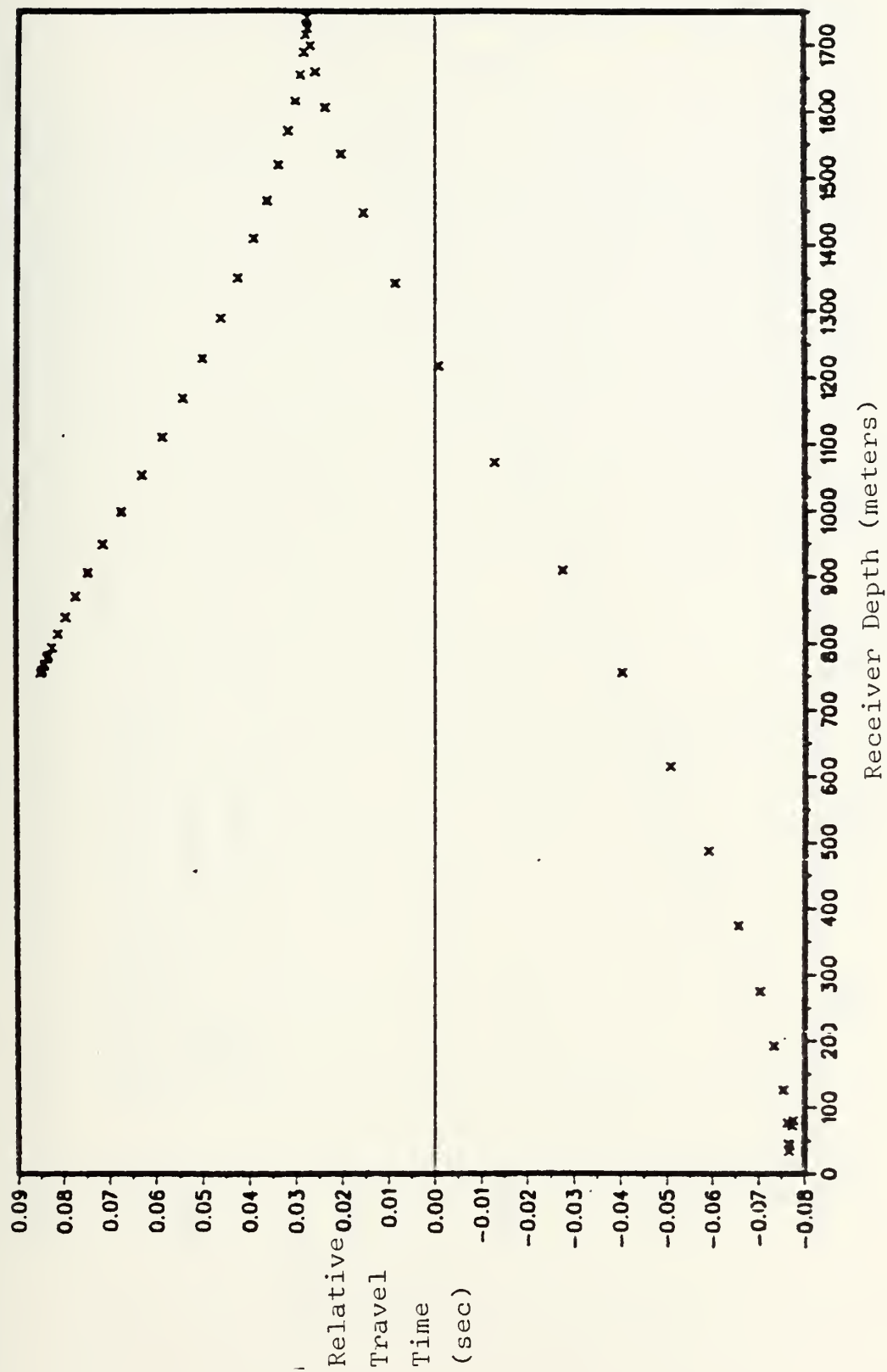


Figure 3.2 Relative Travel Time vs. Depth (380 meter source)



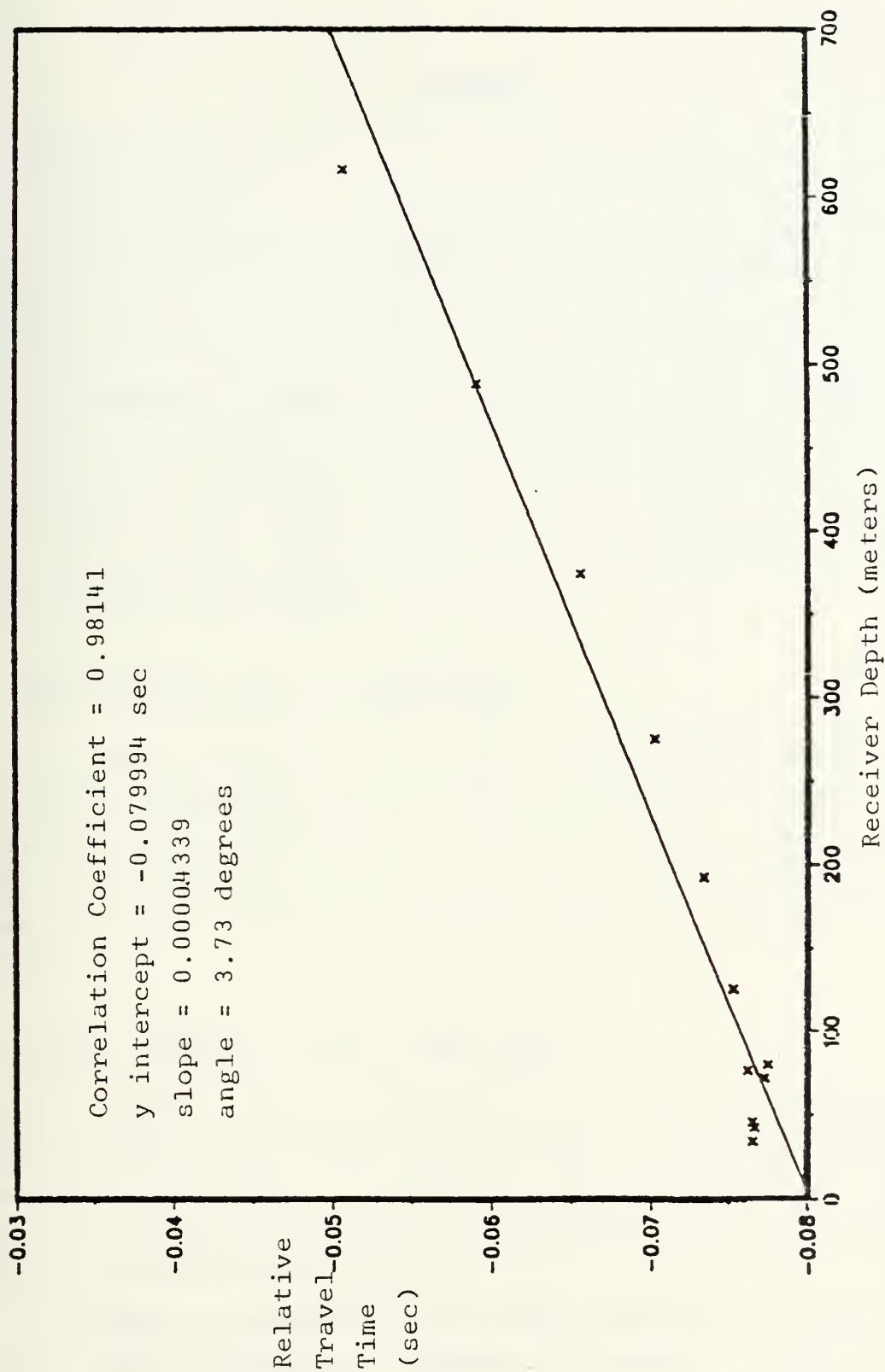


Figure 3.3 Straight Line Approx. of Rel. Trav. Time vs. Depth (source at 380 m)



## IV. RESULTS

### A. EXACT SOLUTION

An exact solution is derived for the beam pattern if the number of receiving hydrophones equal the number of source depths. For example, when the 5 receivers are used to discriminate between 5 source depths (220, 240, 260, 280, and 300 meters) there are 10 equations with 10 unknowns. Figure 4.1 is a plot of the resulting beam pattern with the source at the shallowest depth. Note that because of round-off errors in the 'IMSL' subroutines there is a small value for the resulting beam pattern at the non-energy source depths under investigation.

### B. FOUR DEPTHS WITH TWO RECEIVERS

For source depths at 220, 280, 300, and 320 meters with receiving hydrophones at 100 and 200 meters, there are 8 equations with 4 unknowns. Figure 4.2 is a plot of the resulting beam pattern with a source at the 220 meter depth. Figure 4.3 is the plot for the source at the 280 meter depth.

### C. TWENTY DEPTHS WITH TWO RECEIVERS

For all 20 source depths with receiving hydrophones at 100 and 200 meters, there are 40 equations with 4 unknowns. Figure 4.4 is a plot of the resulting beam pattern for a source at the 220 meter depth. Figure 4.5 is a plot for the source at the 380 meter depth.

In order to determine if the ' $\hat{\theta}$ ' calculated in this case is the best an alternate method is devised. Four source



depths (220, 240, 260 meters, and another source test depth) with the original source at the 220 meter depth and receivers at 100 and 200 meters are used. The beam pattern is calculated each time with a different source test depth substituted for the fourth source depth. The 5 best resulting beam patterns are selected along with the 8 source depths (220, 240, 260 meters, and the 5 test depths which created the 5 best beam patterns). Then, using these 8 source depths, ' $\hat{\theta}$ ' is determined for the two receivers. This ' $\hat{\theta}$ ' is applied to the two receivers and the beam pattern obtained for all 20 source depths.

The resulting beam pattern obtained by this alternate method isn't as good as the beam pattern obtained by using all 20 source depths in the determination of ' $\hat{\theta}$ '.

#### D. TWENTY DEPTHS WITH FIVE RECEIVERS

For all 20 source depths with all 5 receiving hydrophones there are 40 equations with 10 unknowns. Figure 4.6 is a plot of the resulting beam pattern with the source at the 220 meter depth. Figures 4.7, 4.8, and 4.9 are the plots for the source at the 360, 380, and 400 meter depths respectively.

#### E. CONVENTIONAL BEAMFORMER

Figure 4.10 is a plot of the beam pattern for a conventional beamformer using linear phase shifts across the array with the source at the 380 meter depth and the amplitude weights set to unity. All 20 source depths and 5 receiving hydrophones are used. Note that in figure 4.10 that there is less than 1 db discrimination between each of the source depths. Figure 4.11 is the plot obtained for the amplitude weights set to values determined by equation 2.50.





## F. RANGE OF 250 KILOMETERS

The calculations were repeated for a range of 250 km using the same 20 source depths and 5 receiving hydrophones. Figures 4.12 and 4.13 represent the plots of relative travel times versus depth for the 220 and 380 meter source depths respectively. Figure 4.14 represents the straight line approximation of the relative travel times for the 380 meter depth. Note that in this figure the relative travel times are represented by two straight lines; the upper line represents linear phase shifts of the slower travel times for the conventional beamformer while the lower line represents linear phase shifts of the faster travel times. Tables II and III are the straight line interpolations of these slower and faster travel times which correspond to arrival angles of 4.04 and -3.16 degrees respectively. Figures 4.15 and 4.16 are the resulting beam patterns for the conventional beamformer for the slower travel times using unity amplitude weights and linear minimum variance amplitude weights respectively. Figures 4.17 and 4.18 are the beam patterns for the faster travel times.

Figures 4.19, 4.20, 4.21, and 4.22 represent plots of the beam pattern for the source at the 220, 340, 360, and 380 meter depths respectively.



TABLE II

Slower Ray Travel Times For 250 km Range and 380 m Source

<u>Hydrophone Depth</u>	<u>Relative Travel Time</u>
100 meters	-0.09949109 sec.
200 meters	-0.09478615 sec.
300 meters	-0.09008121 sec.
400 meters	-0.08537627 sec.
500 meters	-0.08067133 sec.

TABLE III

Faster Travel Times For 250 km Range and 380 m Source

<u>Hydrophone Depth</u>	<u>Relative Travel Time</u>
100 meters	-0.10100745 sec.
200 meters	-0.10468763 sec.
300 meters	-0.10836781 sec.
400 meters	-0.11204799 sec.
500 meters	-0.11572817 sec.



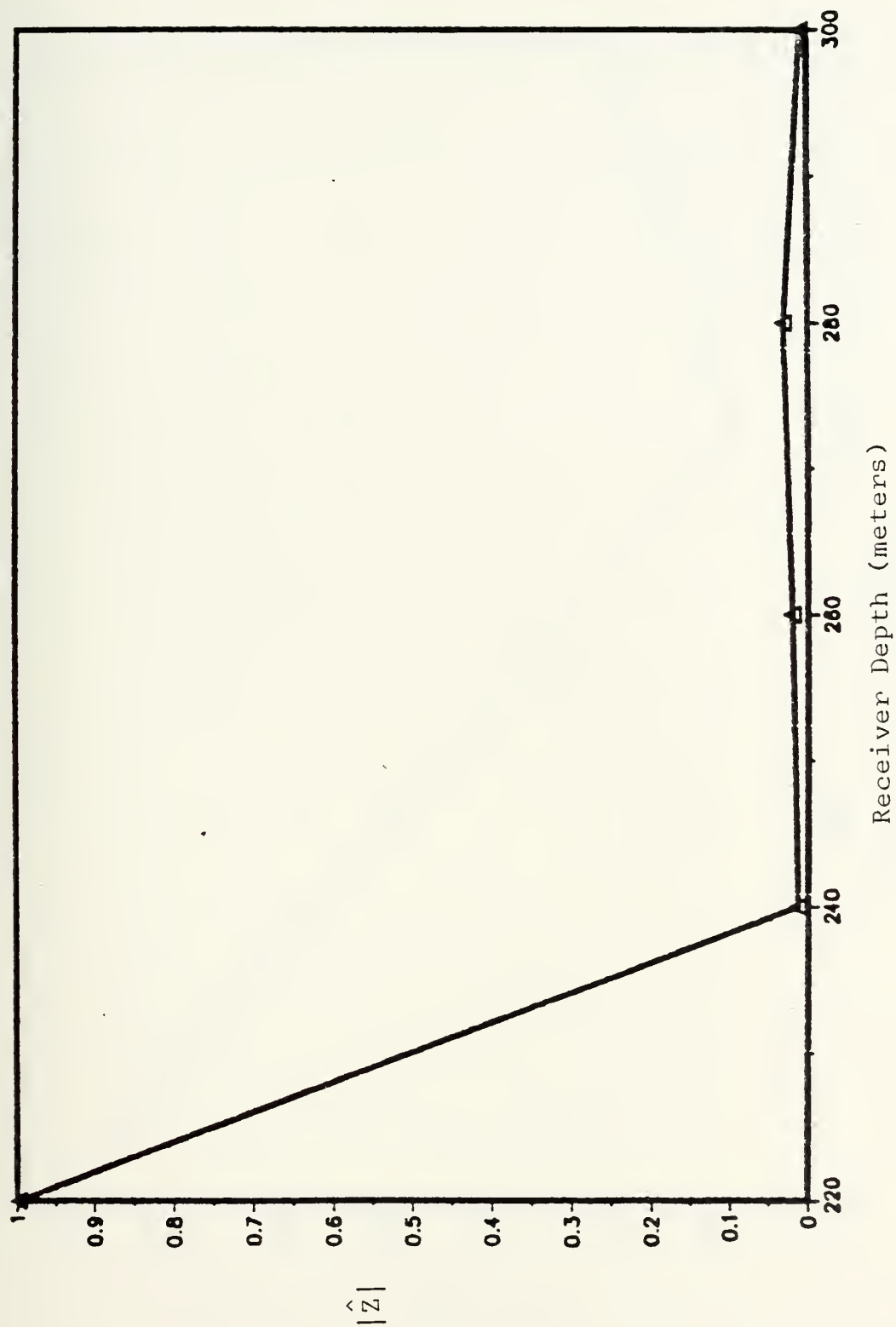


Figure 4.1 Beam Amplitude vs. Source Depth (5 depths 5 receivers)



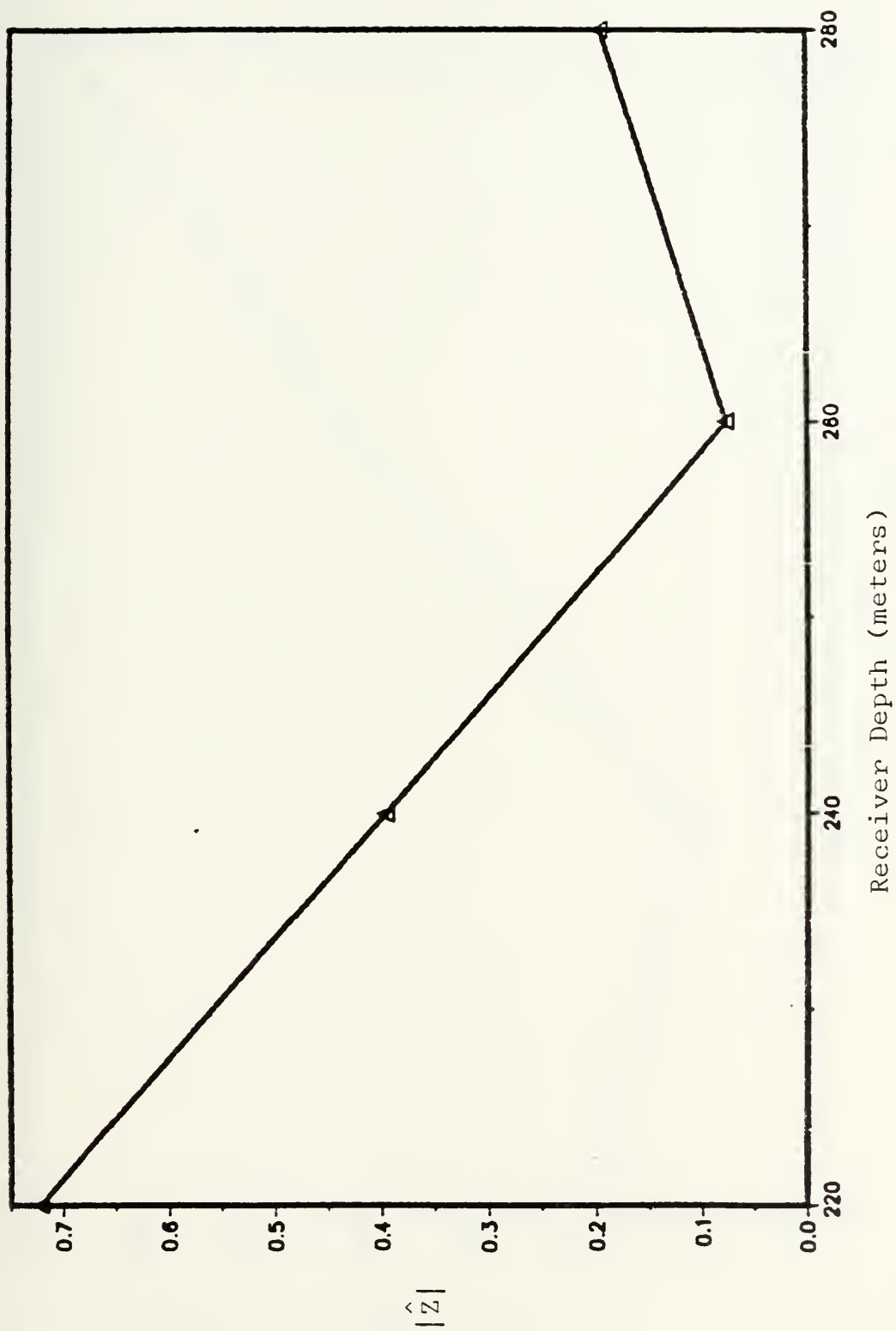


Figure 4.2 Beam Pattern (4 depths 2 receivers, source at 220 m)





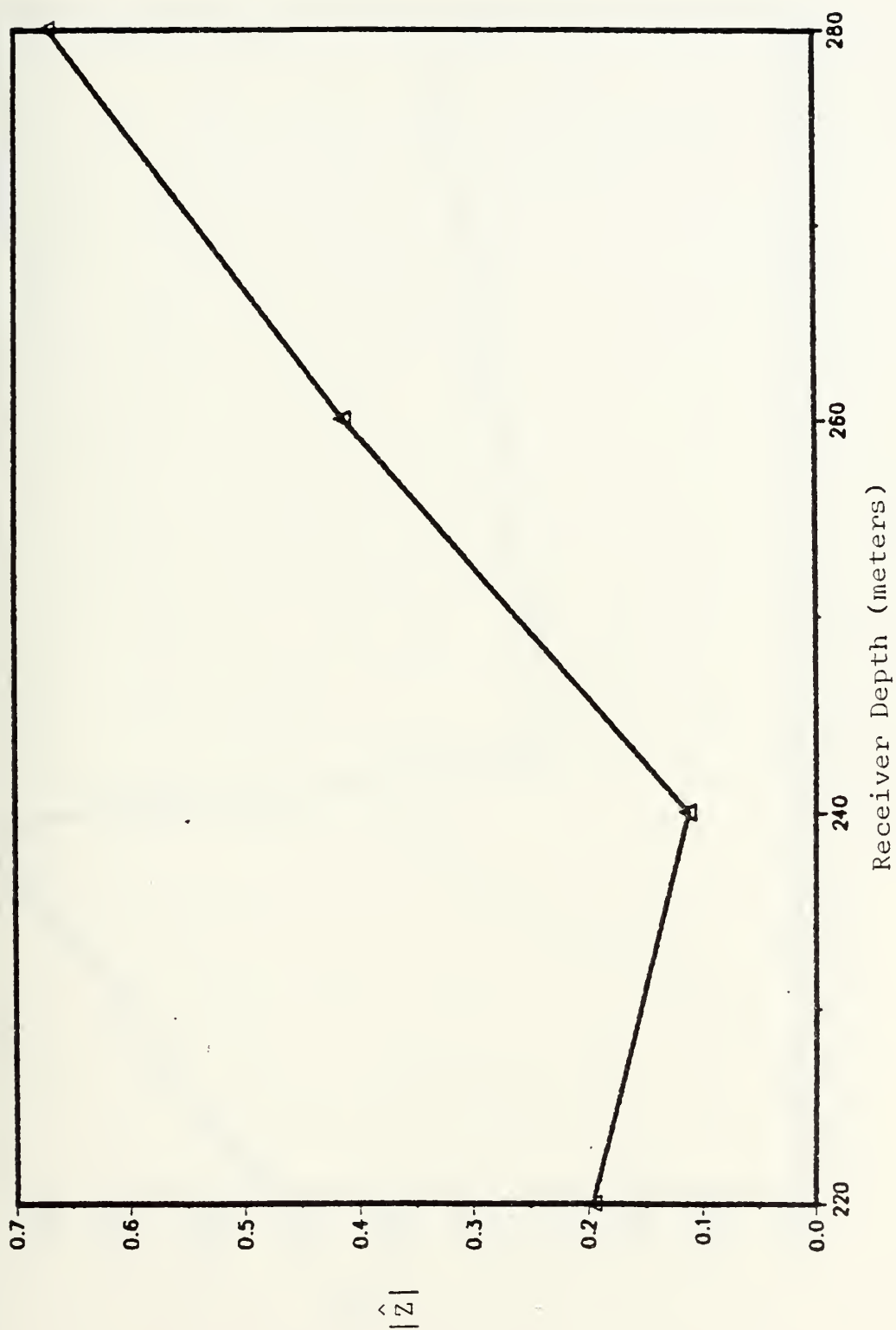


Figure 4.3 Beam Pattern (4 depths 2 receivers, source at 280 m)



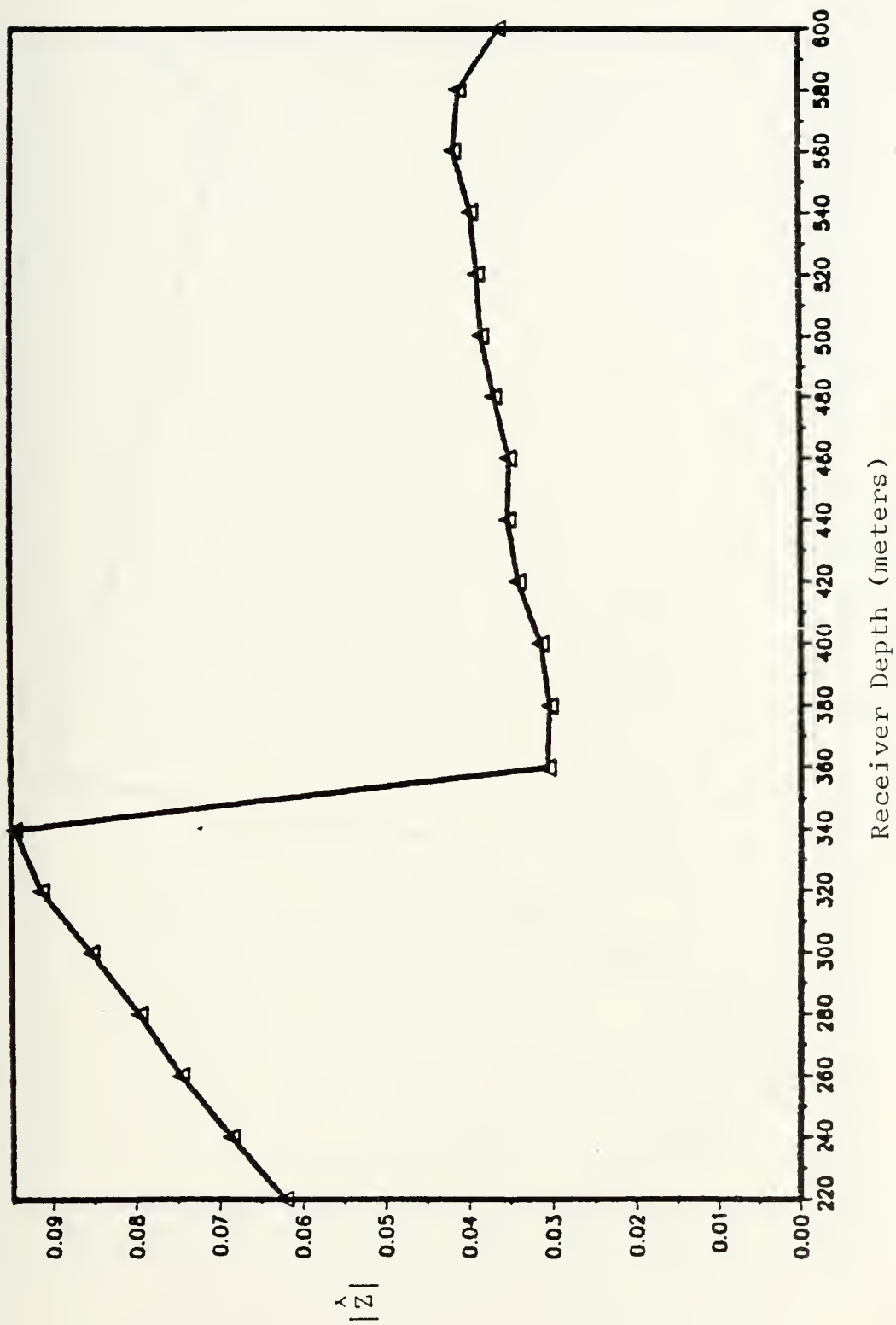


Figure 4.4 Beam Pattern (20 depths 2 receivers, source at 220 m)



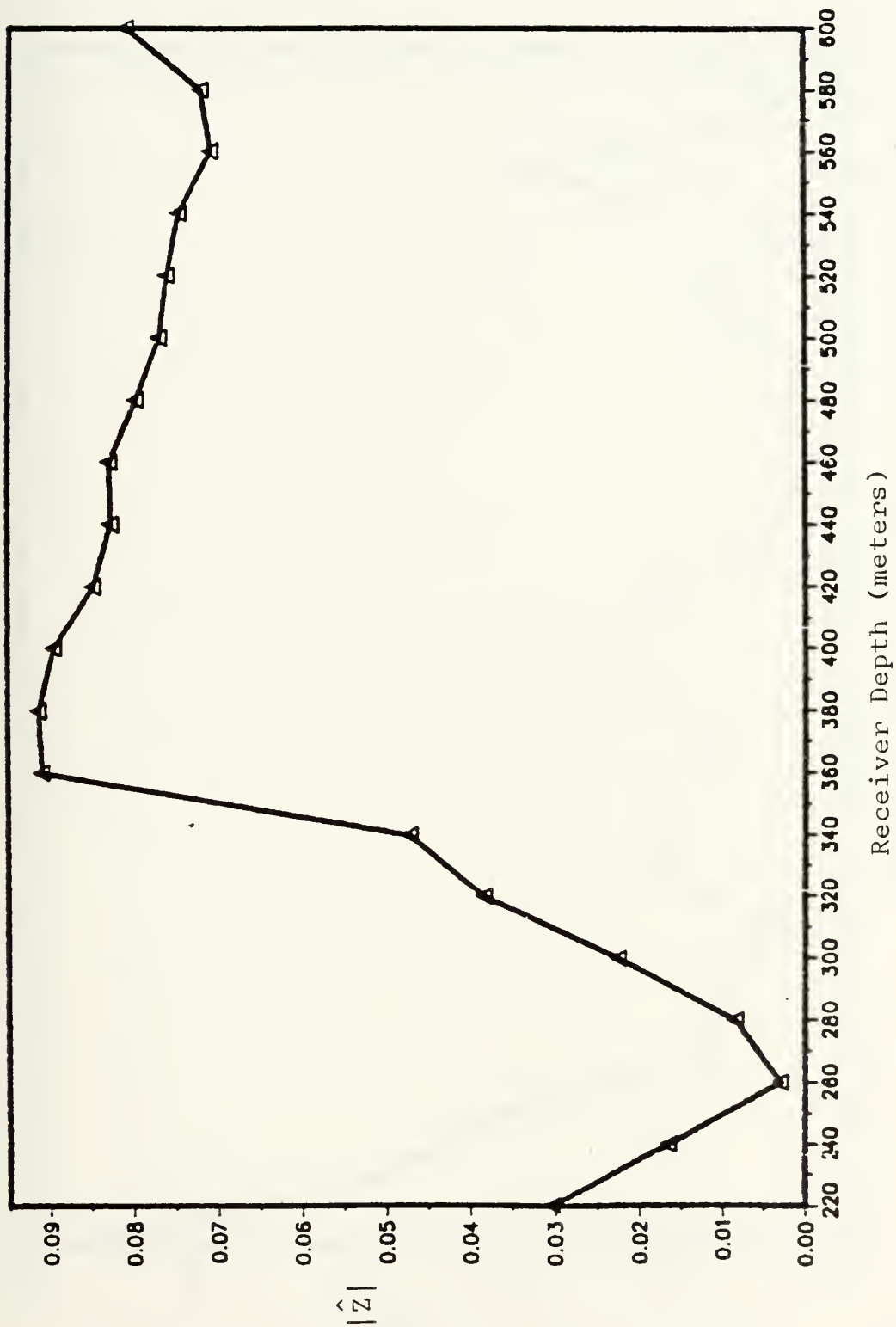


Figure 4.5 Beam Pattern (20 depths 2 receivers, source at 380 m)



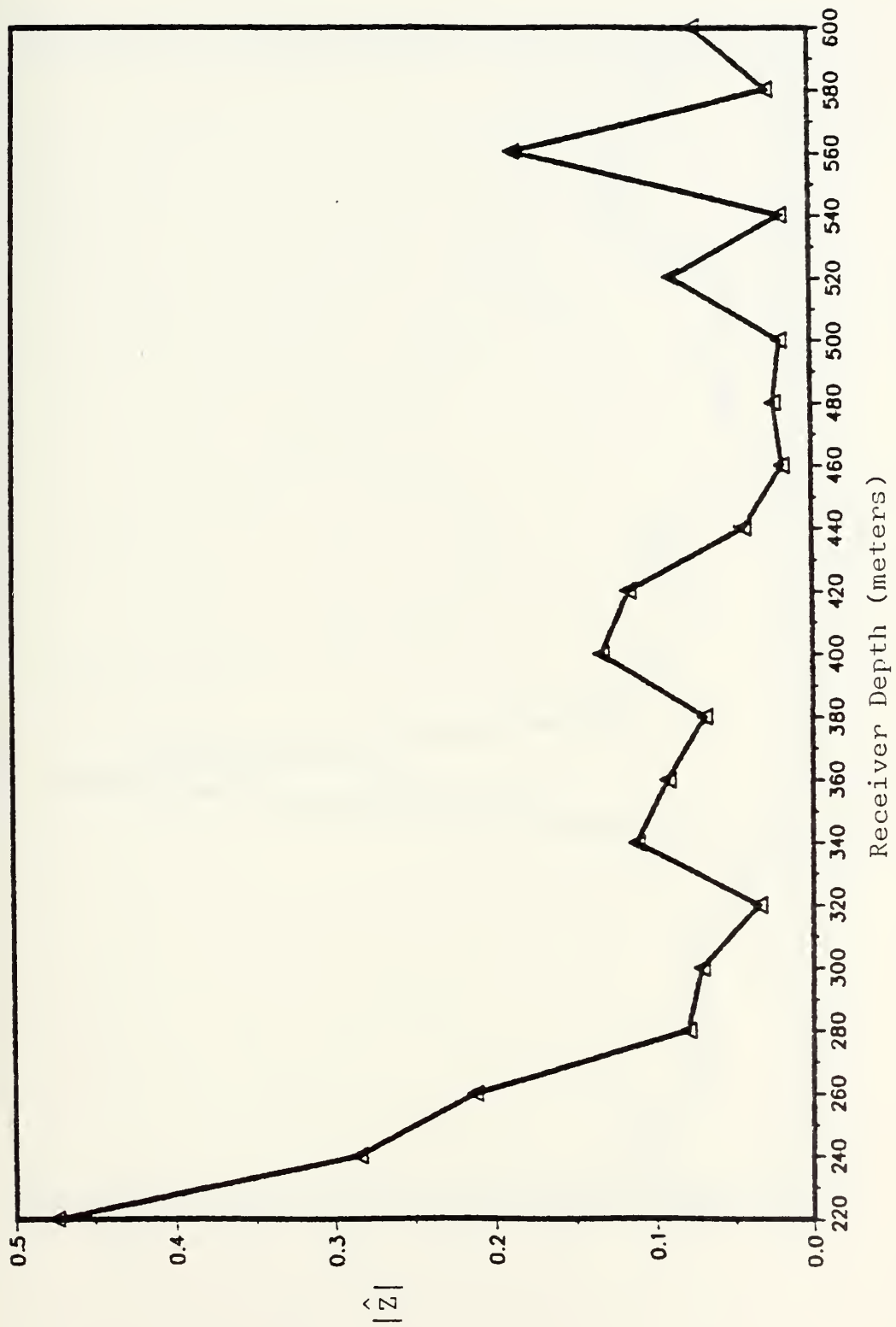


Figure 4.6 Beam Pattern (20 depths 5 receivers, source at 220 m)





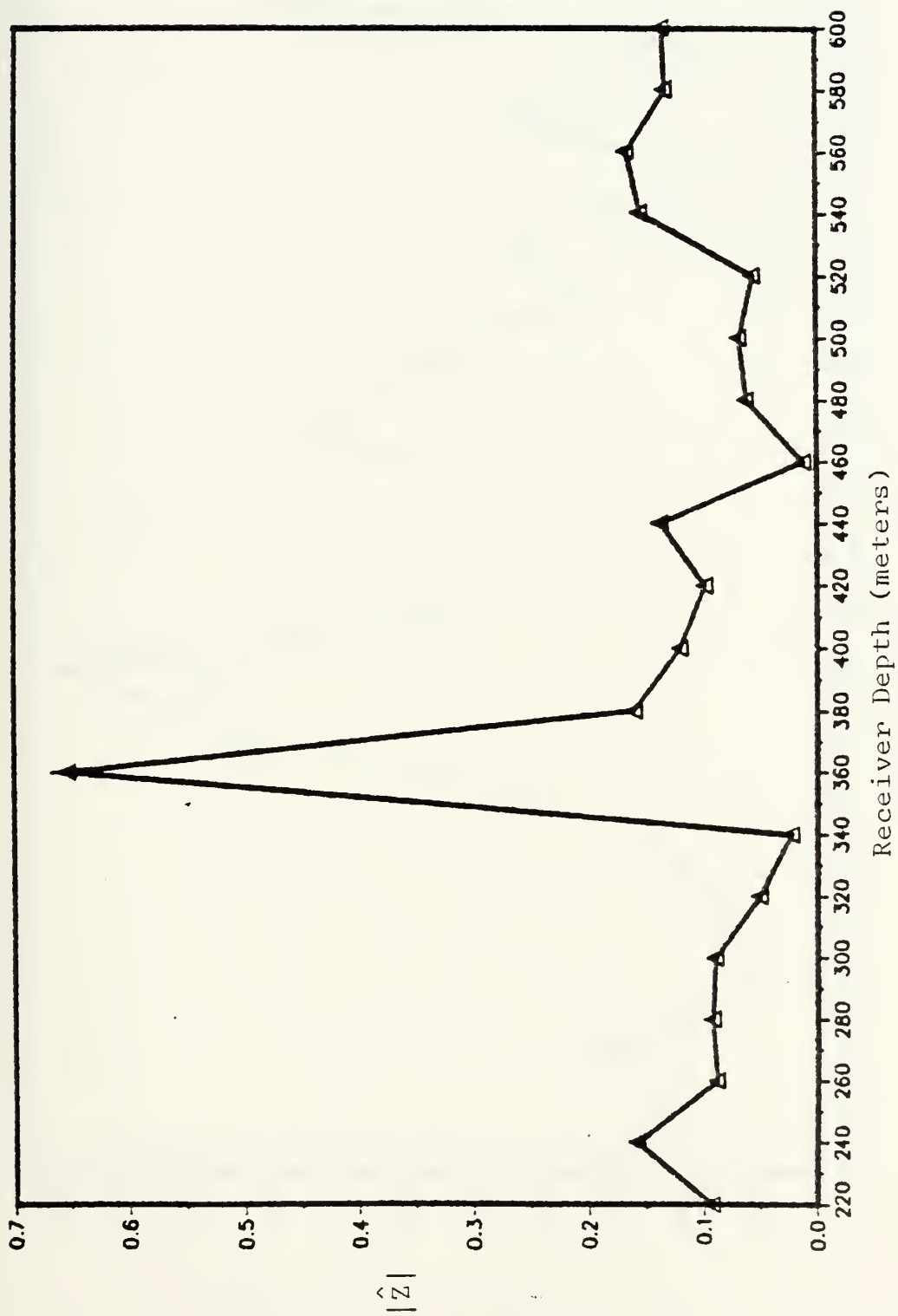


Figure 4.7 Beam Pattern (20 depths 5 receivers, source at 360 m)



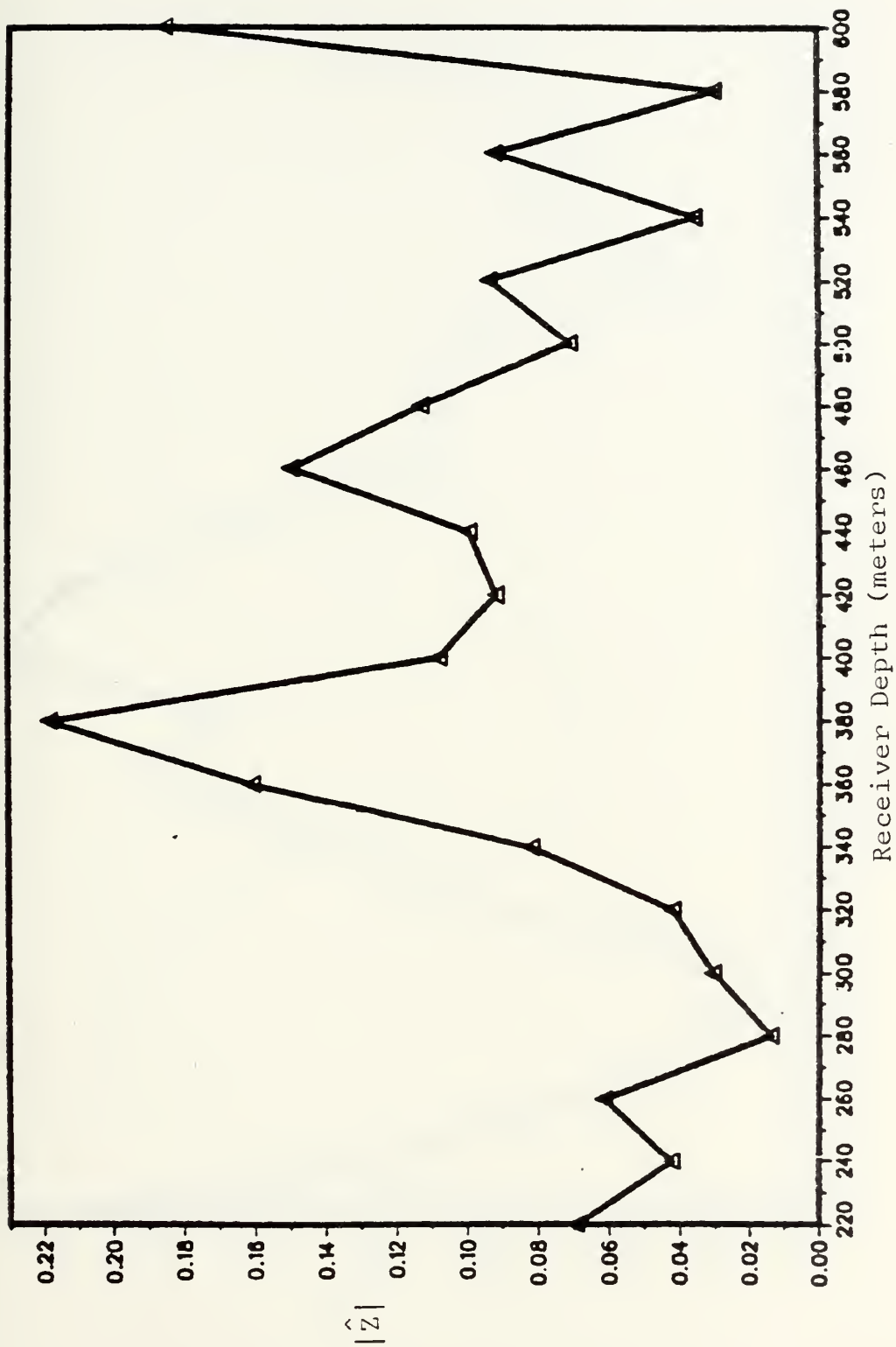


Figure 4.8 Beam Pattern (20 depths 5 receivers, source at 380 m)



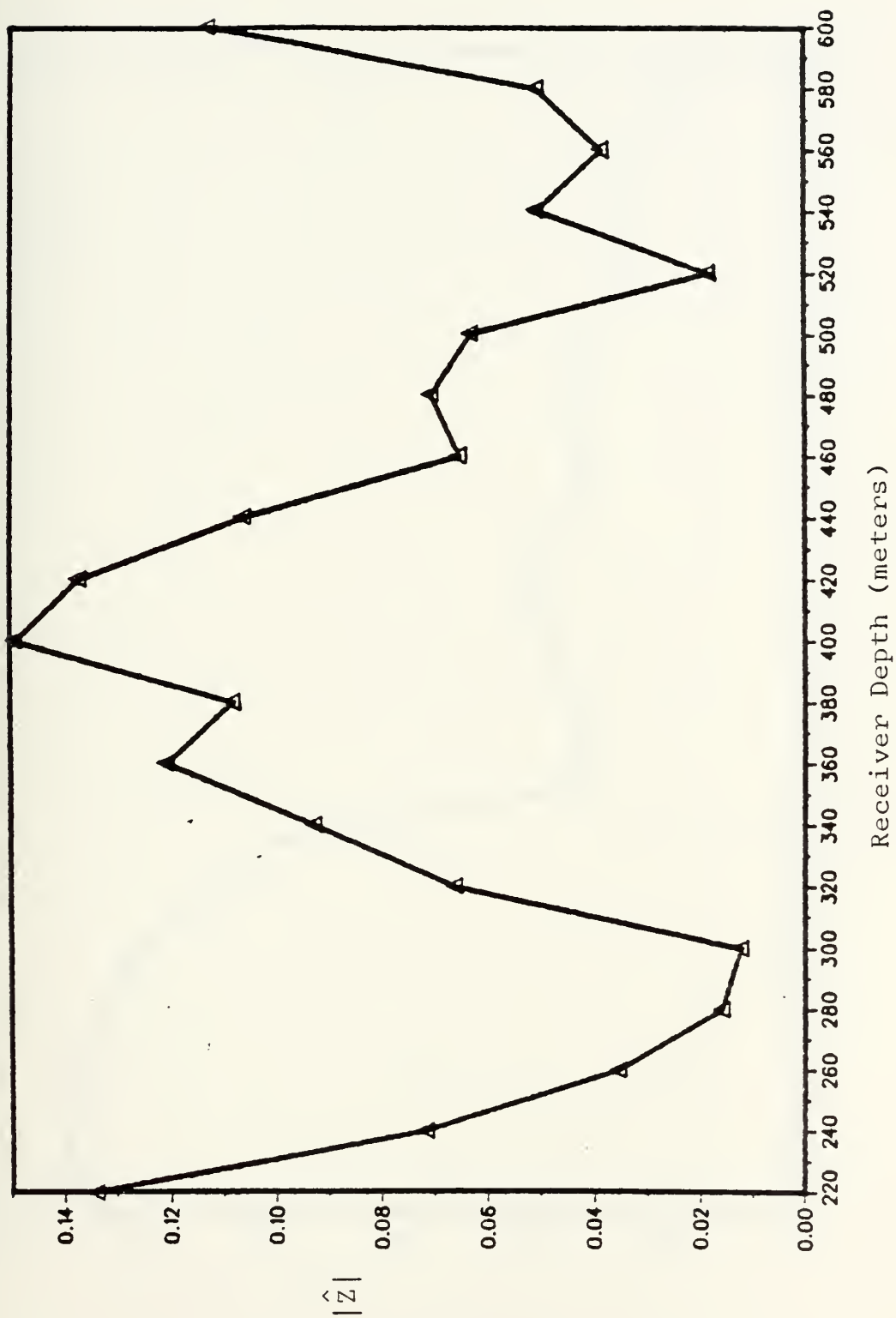


Figure 4.9 Beam Pattern (20 depths 5 receivers, source at 400 m)



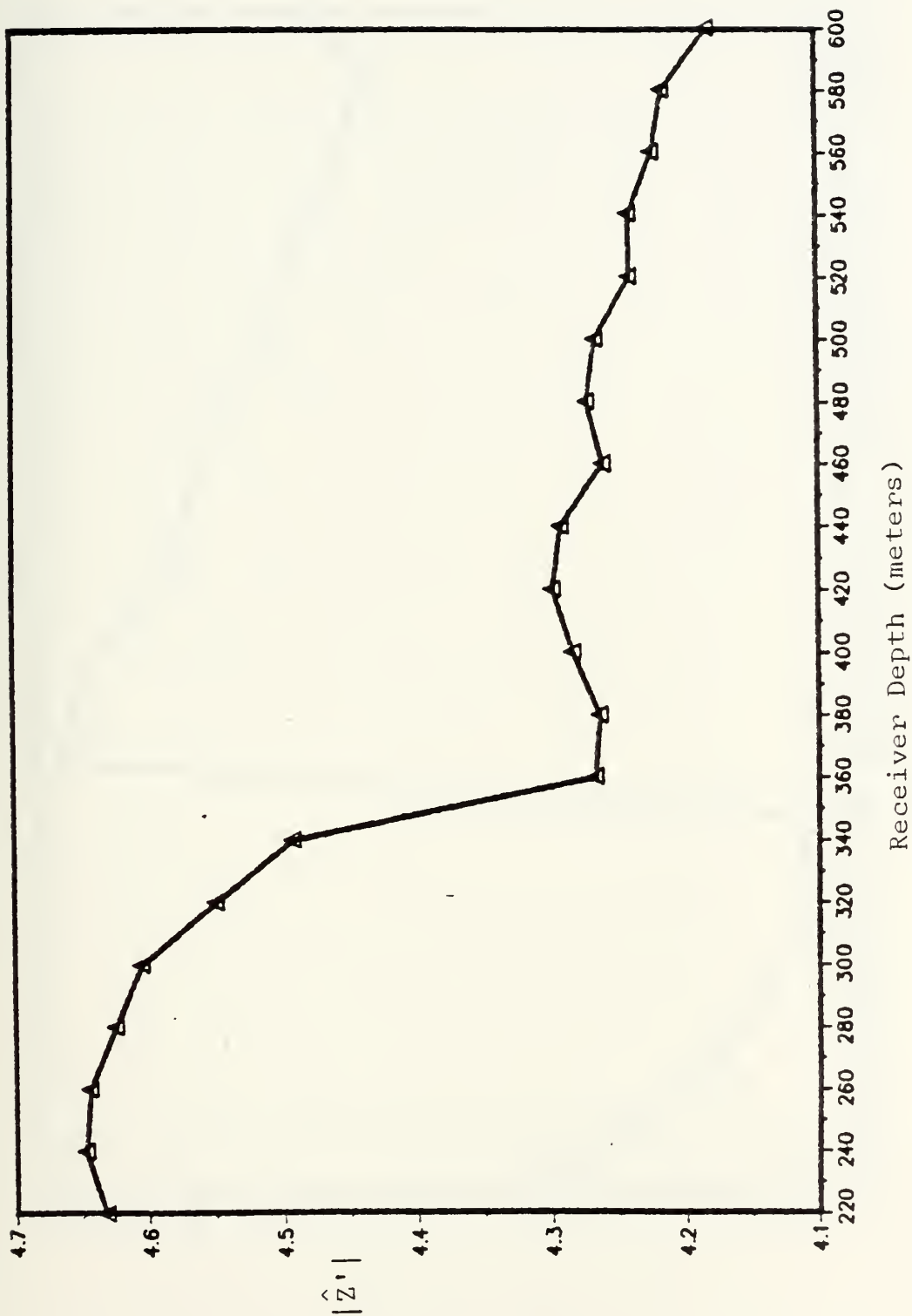


Figure 4.10 Conventional Beam Pattern (source at 380 meters)





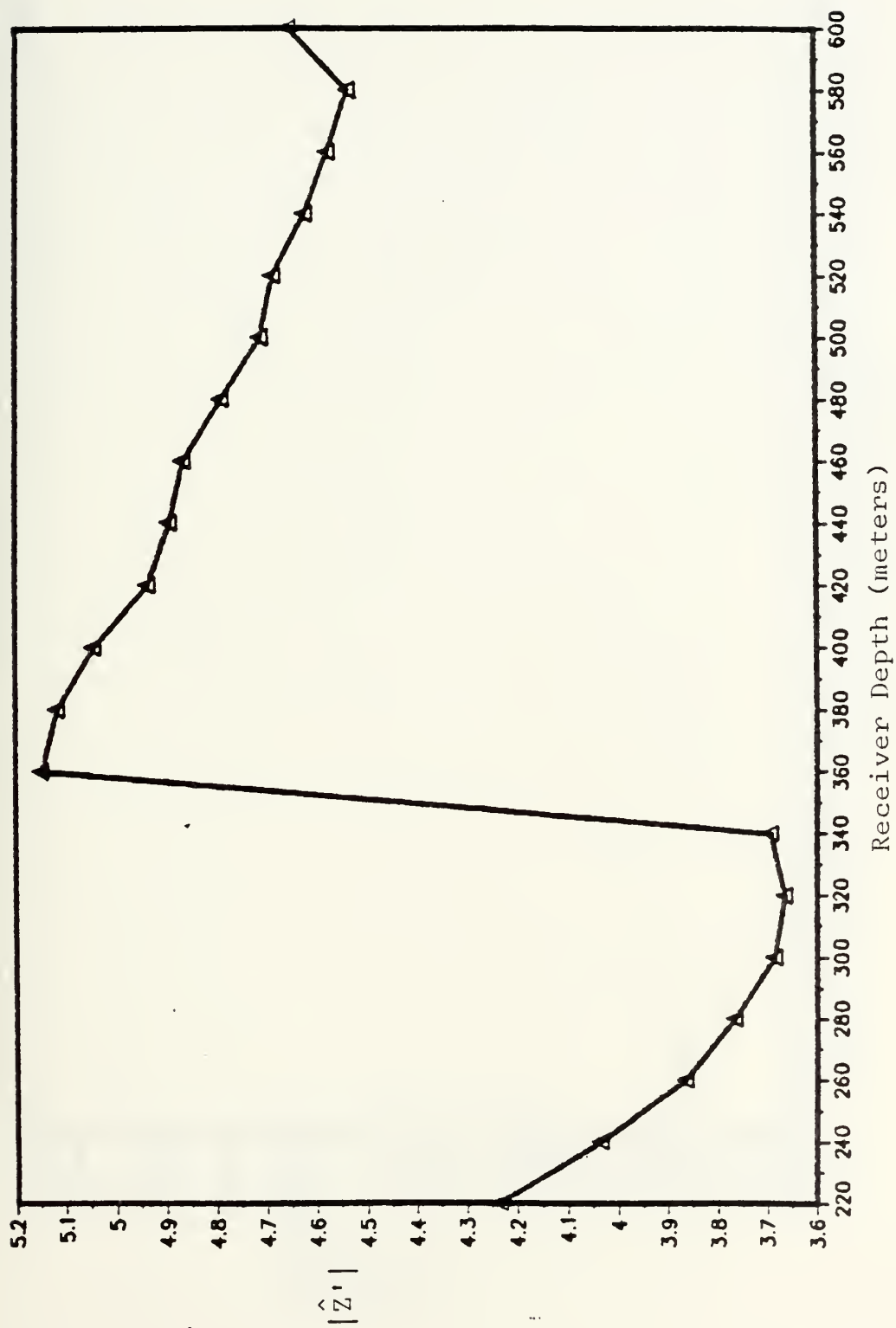


Figure 4.11 Conventional Beam Pattern (non-unity amp. wts. source at 380 m)



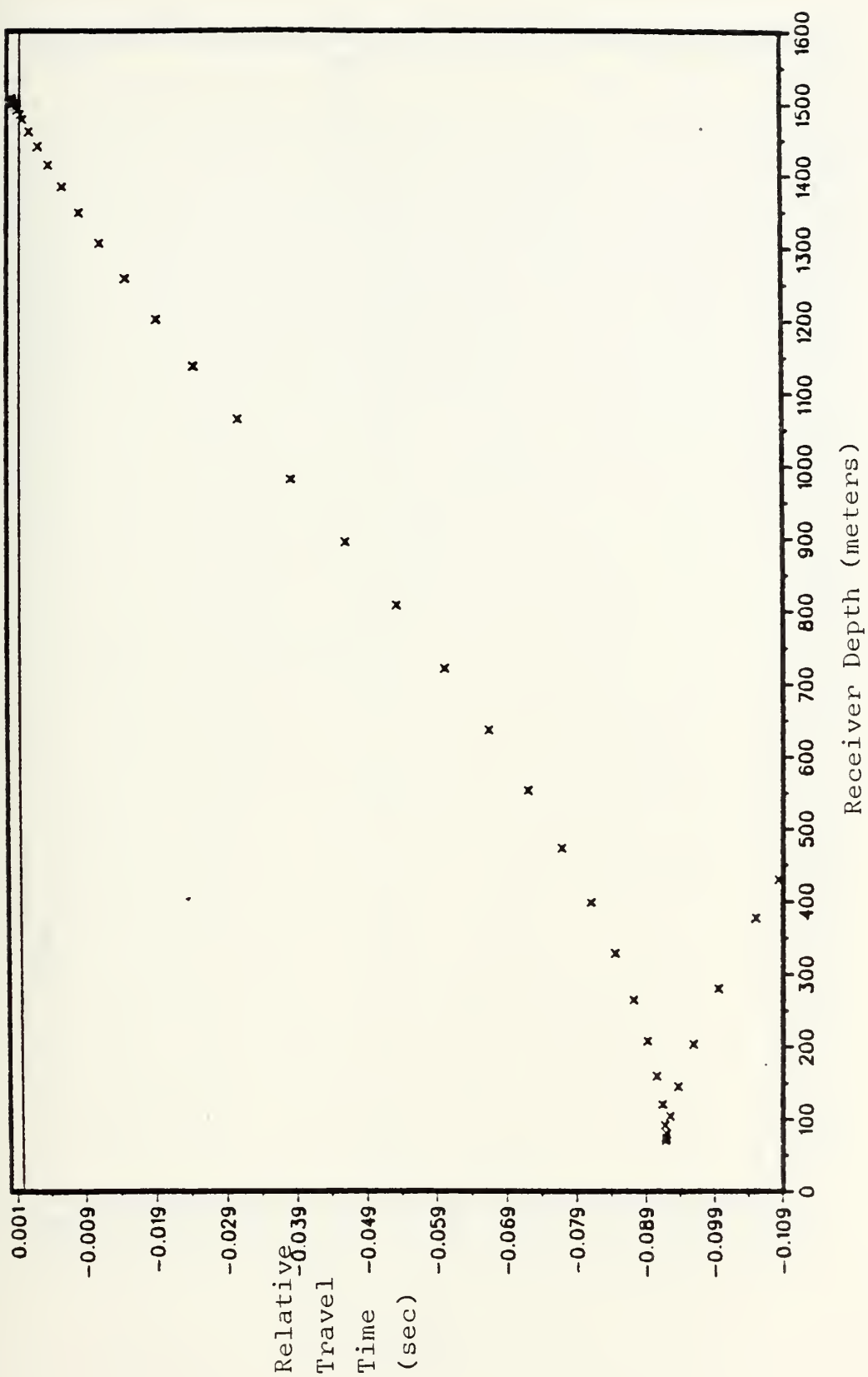


Figure 4.12 Relative Travel Time vs. Depth (range=250 km, source at 220 m)







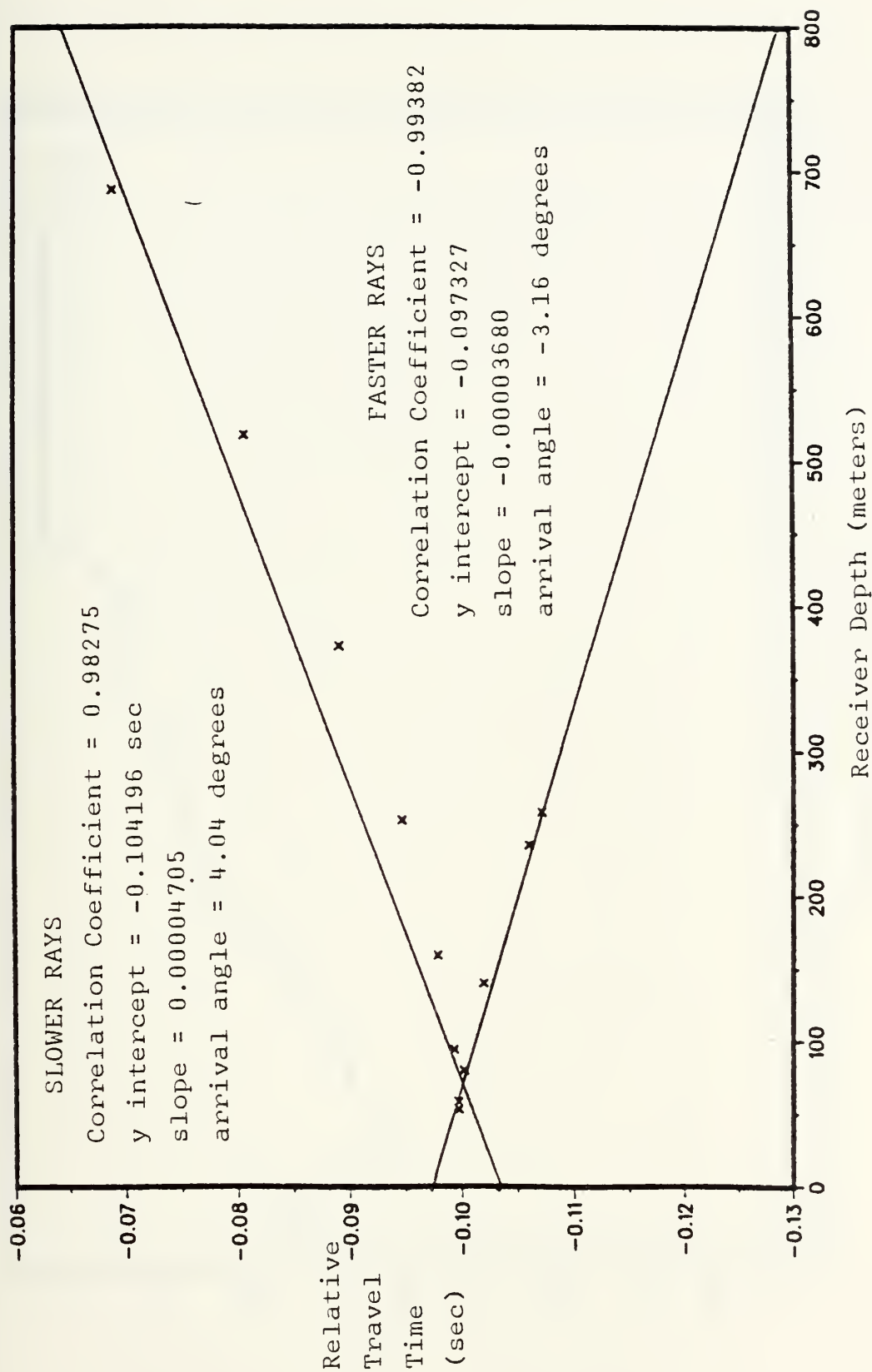


Figure 4.14 St. Line Approx. of Rel. Trav. Time vs. Depth (R=250 km, d=380 m)





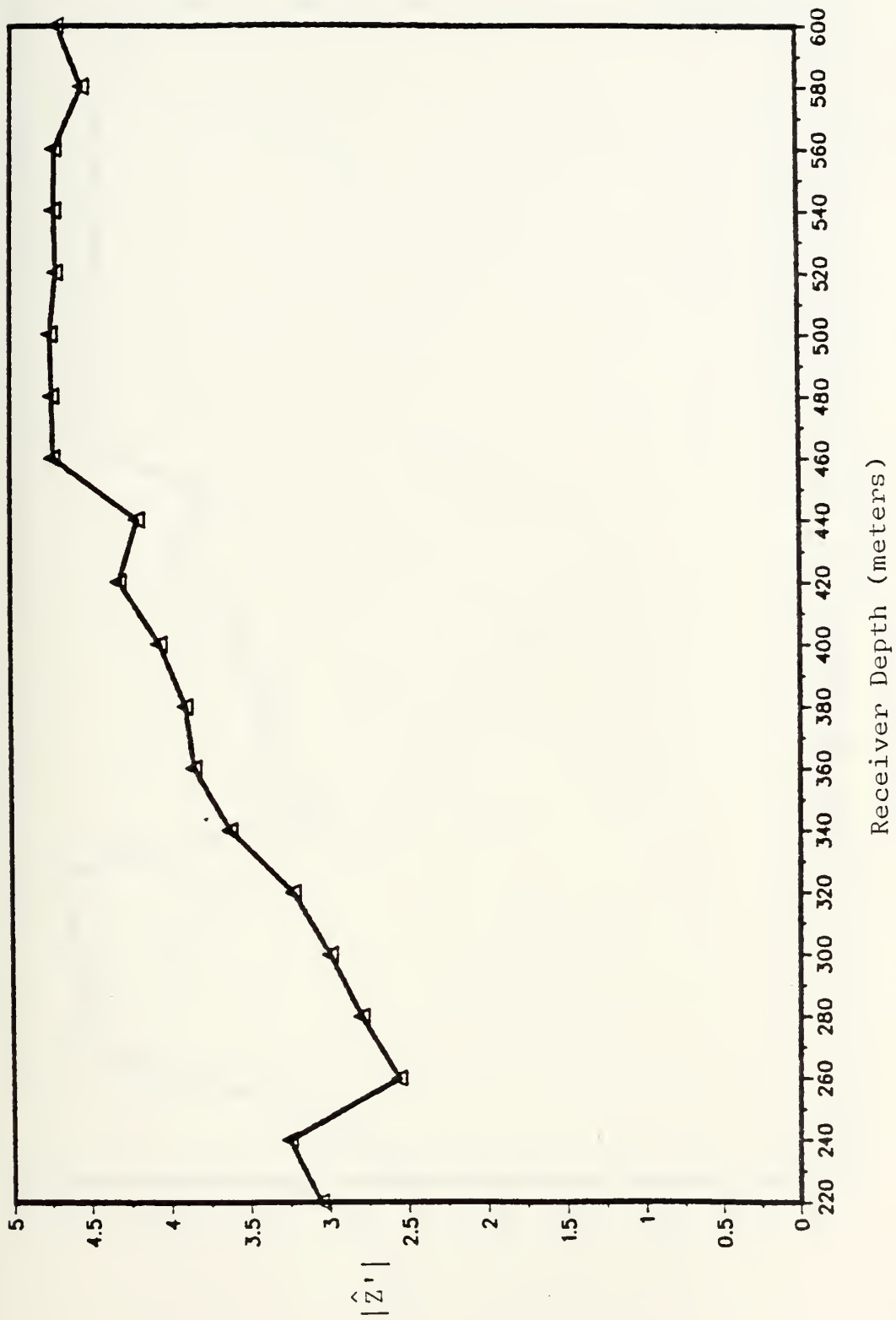


Figure 4.15 Conventional Beam Pattern ( $R=250$  km,  $d=380$  m, slower times,  $a=1$ )



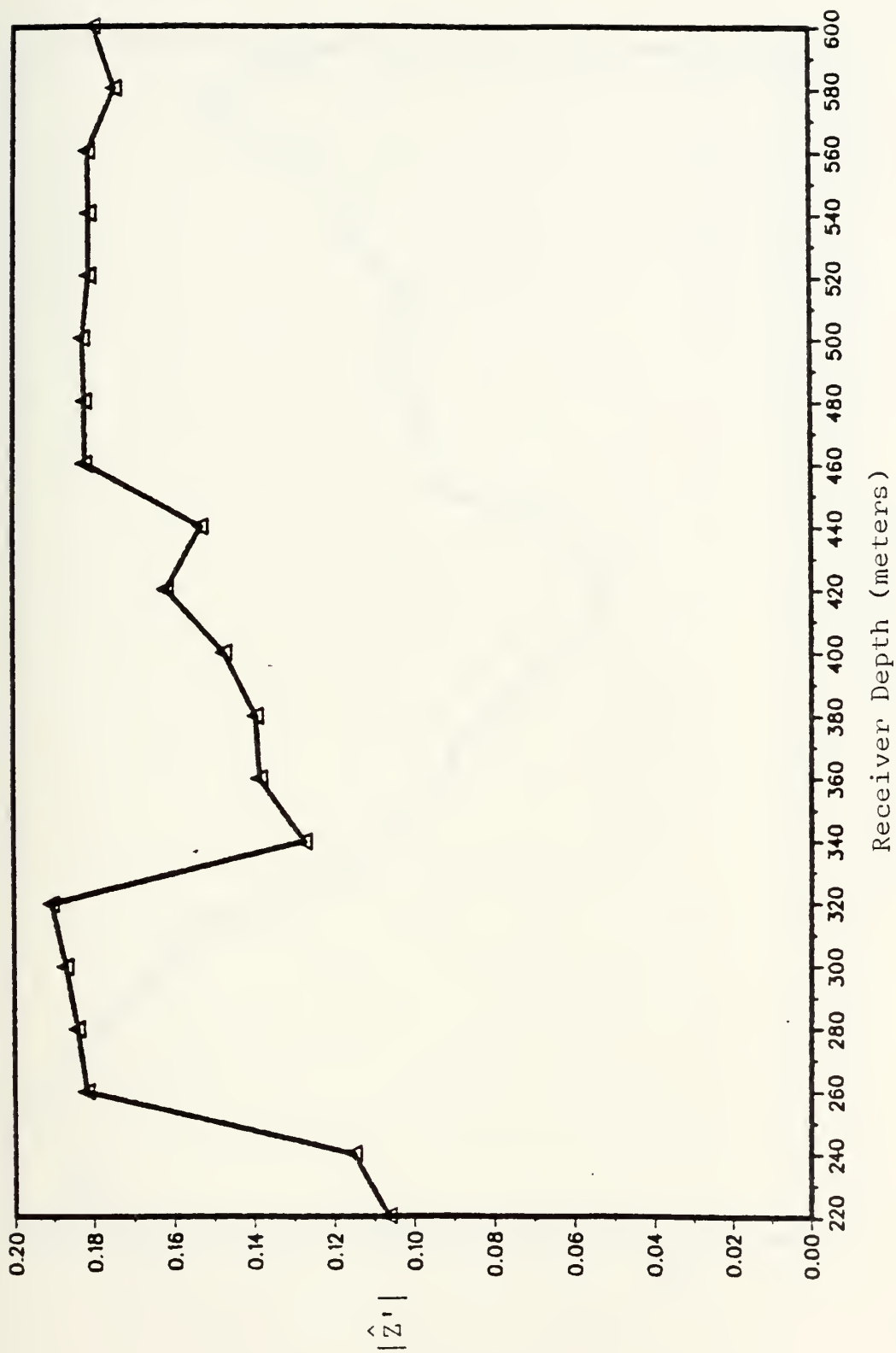


Figure 4.16 Conventional Beam Pattern ( $R=250$  km,  $d=380$  m, slower times, LMV a)



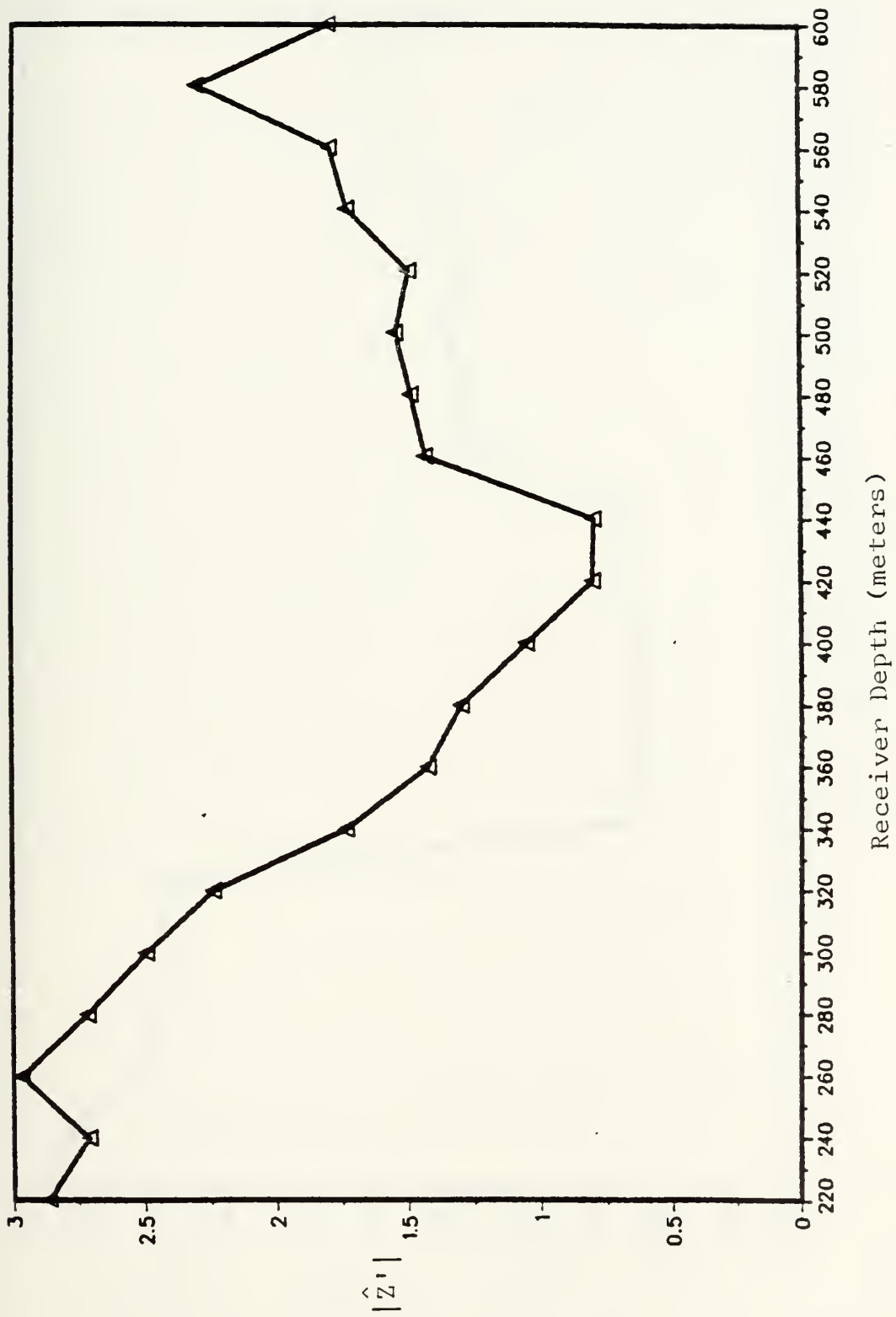


Figure 4.17 Conventional Beam Pattern ( $R=250$  km,  $d=380$  m, faster times,  $a=1$ )



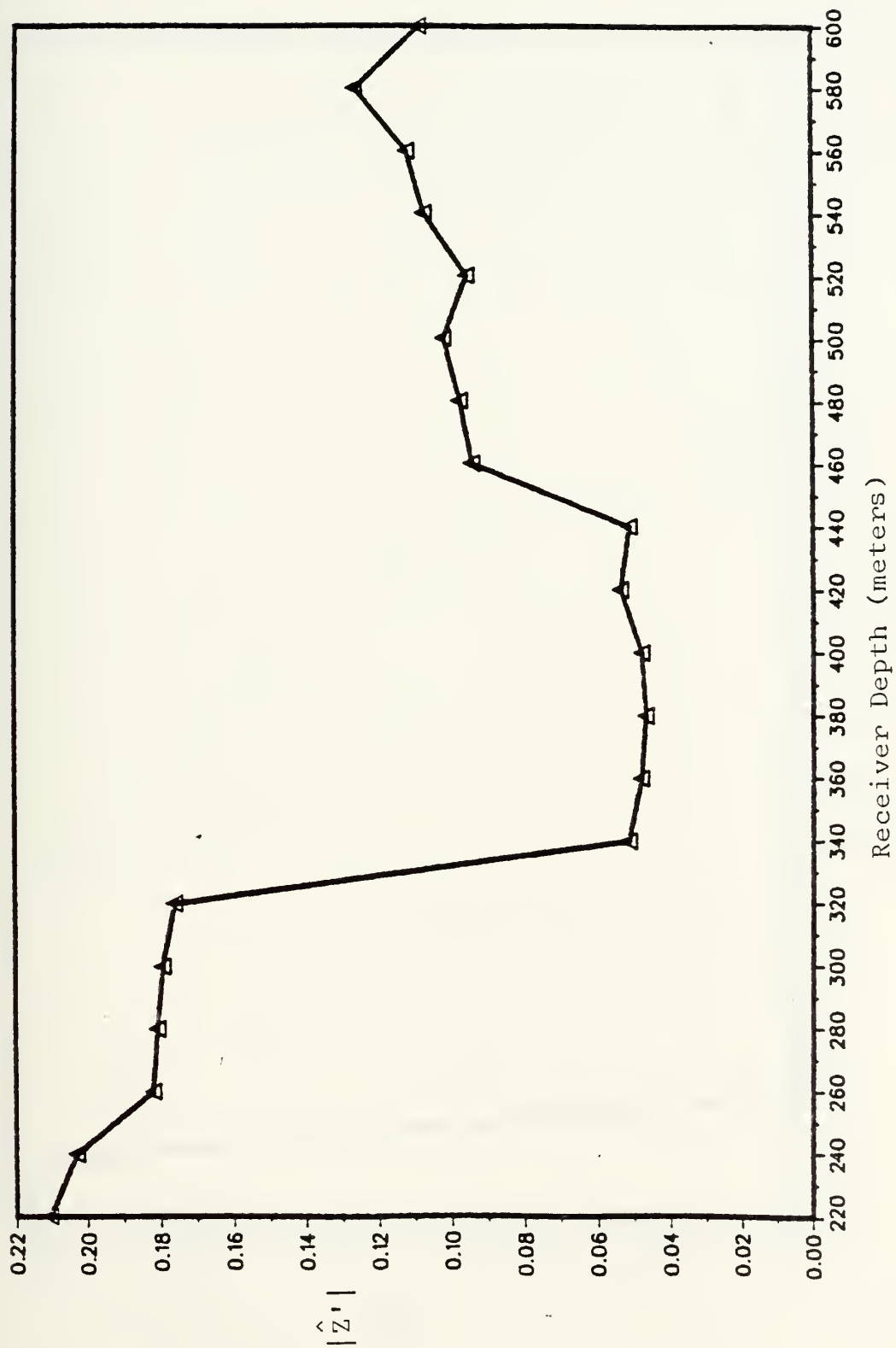


Figure 4.18 Conventional Beam Pattern ( $R=250$  km,  $d=380$  m, faster times, LMV a)





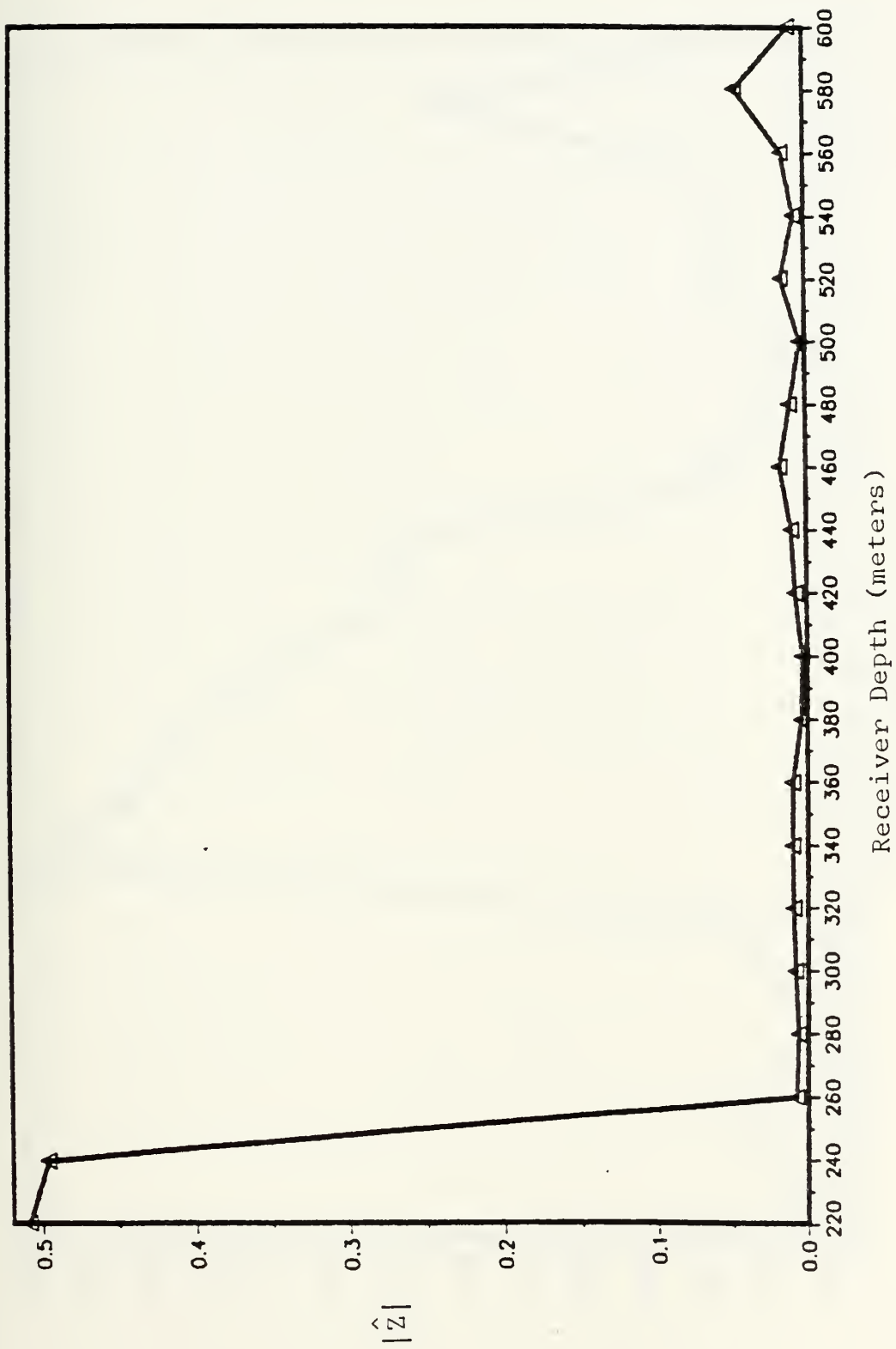


Figure 4.19 Beam Pattern ( $R=250$  km, 20 depths, 5 receivers, source at 220 m)



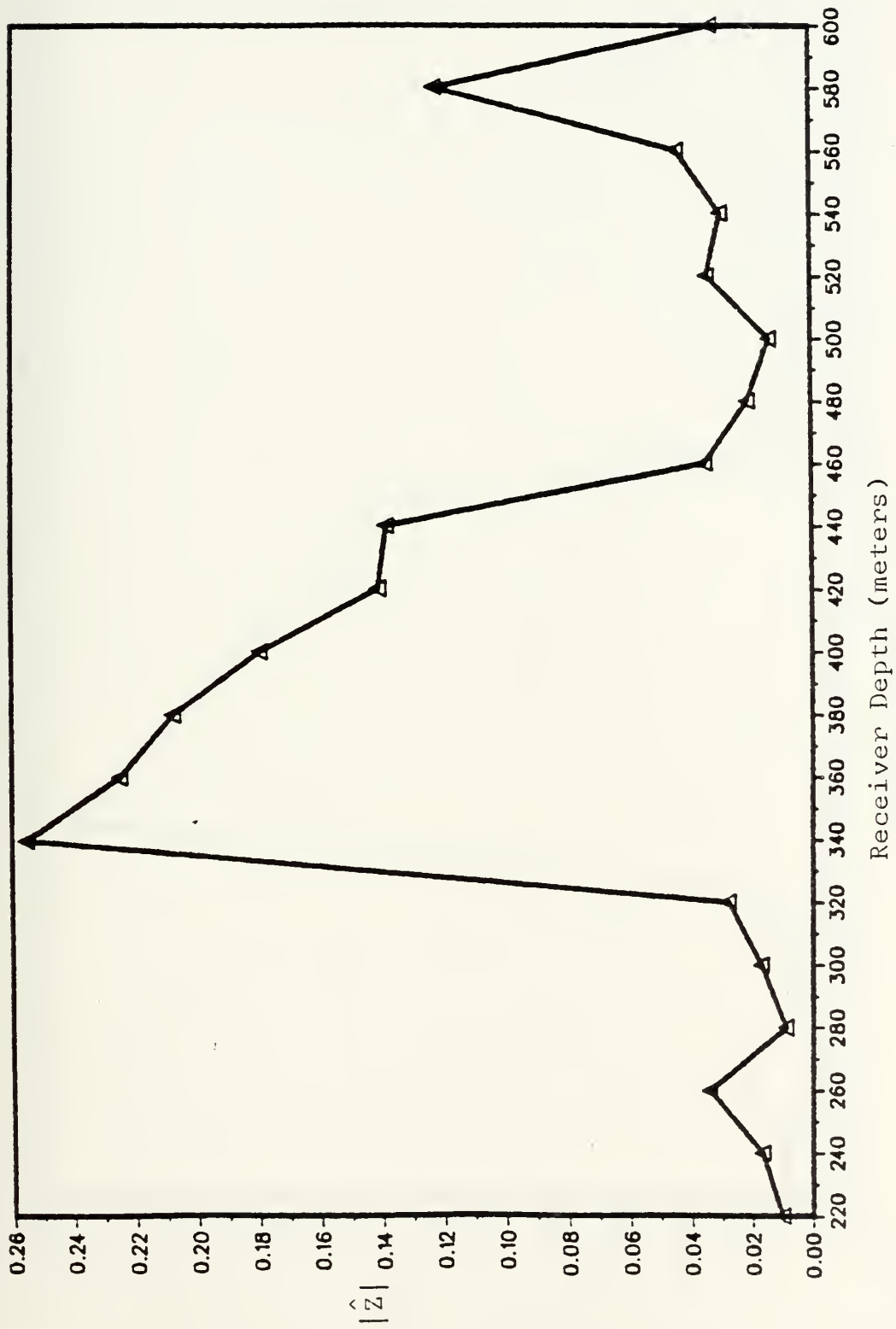


Figure 4.20 Beam Pattern ( $R=250$  km, 20 depths, 5 receivers, source at 340 m)



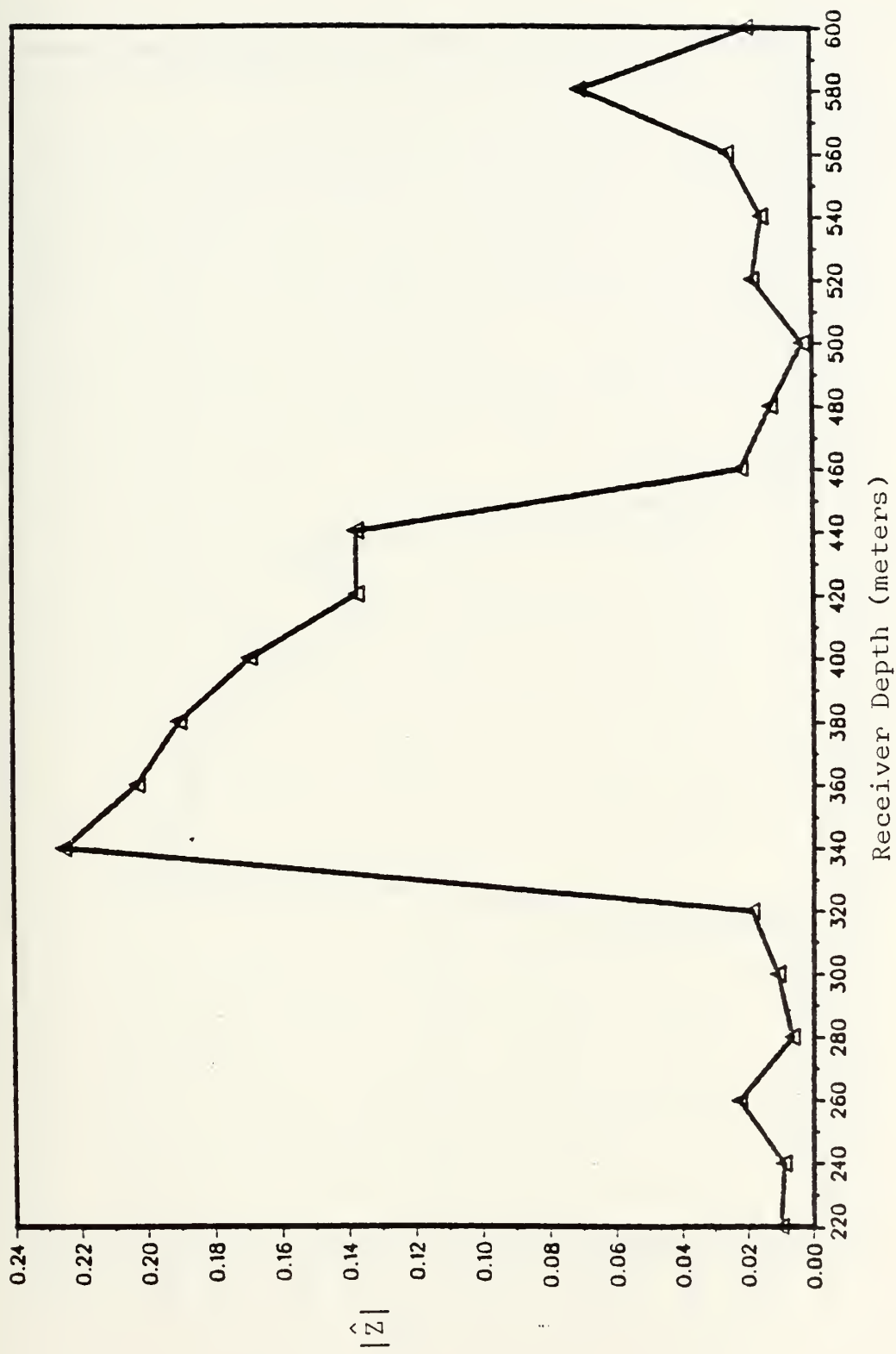


Figure 4.21 Beam Pattern ( $R=250$  km, 20 depths, 5 receivers, source at 360 m)



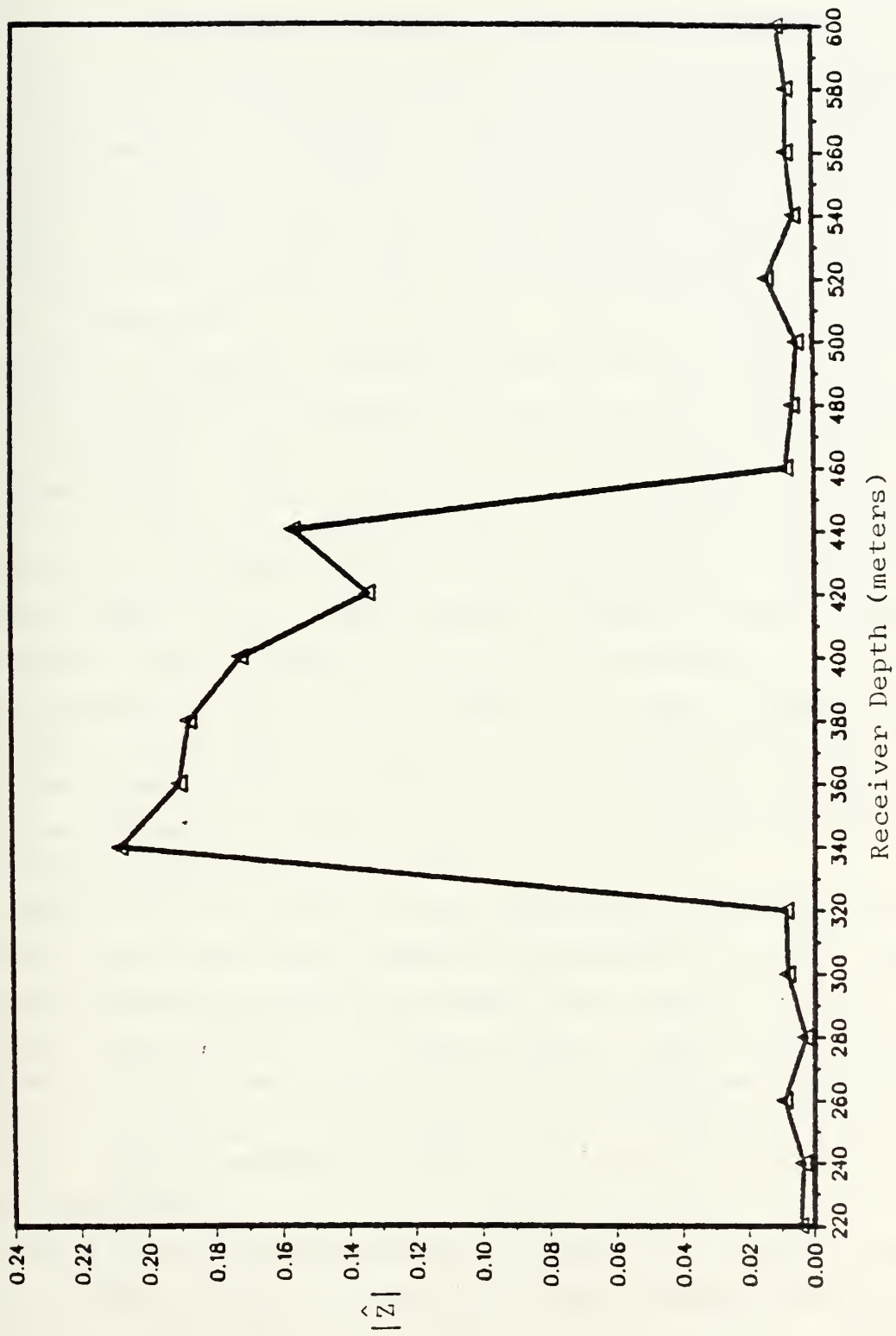


Figure 4.22 Beam Pattern (R=250 km, 20 depths, 5 receivers, source at 380 m)





## V. DISCUSSION OF RESULTS AND RECOMMENDATIONS

For many of the cases examined, the linear minimum variance estimation technique gives high resolution in the "depth beam pattern" for sources at long range. Figures 4.2 and 4.3 represent beam patterns for four depths with two receivers where the main lobe is at the desired source depth with a beamwidth of 30 meters. For 20 depths with 2 receivers the system is highly overdetermined and the beam pattern portrayed in figure 4.4 has its main lobe 80 meters from the source depth. At the source depth the strength of the beam pattern is 3.6 db down. For figure 4.5 the main lobe is on the source depth with a beamwidth of 250 meters.

For all 20 source depths with 5 hydrophone receivers the results range from a beam pattern having its main lobe on the source depth (figure 4.7) with a beamwidth of 20 meters and a secondary lobe 12.1 db down to a beam pattern having its main lobe 40 meters from the source depth (figure 4.22). For the beam pattern in figure 4.22 the width of the main lobe is 80 meters and the strength of the beam pattern at the source depth is 0.8 db down.

When using the conventional beamformer, the beam pattern results with amplitude weights determined by the linear minimum variance estimation method were superior to the beam pattern determined by using unity amplitude weights. However in all cases the conventional beamformer was significantly inferior to the beam pattern determined by the linear minimum variance estimation technique in terms of depth resolution.

The linear minimum variance estimation technique could not be used for all ranges with the chosen sound speed profile. For ranges of 187 km and 235 km the relative



travel times produce an 'A' matrix which is so ill-conditioned that the 'IMSL' subroutine 'LINV2F' cannot determine an accurate inverse of 'A<sup>T</sup>A'. .

There is sufficient evidence from this initial investigation that the linear minimum variance estimation technique applied to a long linear vertical array can yield high resolution depth information about passive sources at very long ranges. However, further investigation is needed before any field tests are in order. Recommendations for further research are:

1. The use of a more realistic speed of sound profile, preferably one actually characteristic of the 'SCFAR' channel.
2. The use of more hydrophones in the vertical array. As more receivers are used the beam pattern should approach the desired beam pattern more closely (a less overdetermined system should yield smaller total minimum mean square error).
3. Investigation into causes for the peak response falling at other than the desired depth and techniques for correcting this problem when using the linear minimum variance estimation technique.
4. Alternate assumption for choosing the ray paths to include in the 'A' matrix. For example one might choose the rays with the greatest intensity instead of the shortest travel time.
5. The possibility of estimating range as well as the depth of a passive source with linear minimum variance estimation techniques.

In conclusion, the linear minimum variance estimation technique of beamforming was significantly superior to the conventional beamformer. High resolution depth information about passive sources at long ranges is provided.



APPENDIX A  
RELATIVE TRAVEL TIME CALCULATION

This program calculates the relative travel times from each source depth to the horizontal range and the depth of the ray at this range. To save paper, only the two shallowest source depths are listed.













THE0C570  
THE0C580  
THE0C590  
THE0C600  
THE0C610  
THE0C620  
THE0C630  
THE0C640  
THE0C650  
THE0C660  
THE0C670  
THE0C680  
THE0C690  
THE0C700  
THE0C710  
THE0C720  
THE0C730  
THE0C740  
THE0C750  
THE0C760  
THE0C770  
THE0C780  
THE0C790  
THE0C800  
THE0C810  
THE0C820  
THE0C830  
THE0C840  
THE0C850  
THE0C860  
THE0C870  
THE0C880  
THE0C890  
THE0C900  
THE0C910  
THE0C920  
THE0C930  
THE0C940  
THE0C950  
THE0C960  
THE0C970  
THE0C980  
THE0C990  
THE0C1000  
THE0C1010  
THE0C1020  
THE0C1030  
THE0C1040  
THE0C1050  
THE0C1060  
THE0C1070  
THE0C1080  
THE0C1090  
THE0C1100  
THE0C1110  
THE0C1120  
THE0C1130  
THE0C1140  
THE0C1150  
THE0C1160  
THE0C1170  
THE0C1180  
THE0C1190  
THE0C1200  
THE0C1210  
THE0C1220  
THE0C1230  
THE0C1240  
THE0C1250  
THE0C1260  
THE0C1270  
THE0C1280  
THE0C1290  
THE0C1300  
THE0C1310  
THE0C1320  
THE0C1330  
THE0C1340  
THE0C1350  
THE0C1360  
THE0C1370  
THE0C1380  
THE0C1390  
THE0C1400  
THE0C1410  
THE0C1420  
THE0C1430  
THE0C1440

```
CRAN=(C1*Y1)/(G*X1)
RAN=RAN+CRAN
R11=RAN
IF (RANGE-RAN)12,12,14
IF HORIZONTAL RANGE OF ARC 1 AND 2 IS LESS THAN THE TOTAL
RANGE THEN CONTINUE PROGRAM, OTHERWISE EXIT TO 12
CALCULATE SUMMED TRAVEL TIME
C1=C1+G*CDEF
DEPTH=DEPTH+CDEP
D2=DEPTH
TIME=TIME+((-1.0C0/G)*DLOG((DTAN(P))/(DTAN(P+(F2/2.0D0))))))
ADD ARC 3
T2=TIME
CDEP=-CDEP
X2=((CDEP*G)/C1)+1.0D0
F2=-DARCSIN(X2)
Y2=DSIN(F2)
DRAN=(C1*(-Y2))/G
RAN=RAN+CRAN
R2=RAN
IF (RANGE-RAN)21,21,22
ADC ARC 4
C1=C1+G*CDEF
DEPTH=DEPTH+CDEP
D3=DEPTH
TIME=TIME+((-1.0C0/G)*DLOG((DTAN(P+(F2/2.0D0))))/(DTAN(P))))
T3=TIME
G=-0.017D0
X1=X2
Y1=Y2
CDEP=(C1*(1.0D0-X1))/(G*X1)
ERAN=C1*Y1/(G*X1)
F1=DARCSIN(Y1)
RAN=RAN+ERAN
R3=RAN
IF (RANGE-RAN)41,41,42
ADD ARC 5
C1=C1+G*CDEF
DEPTH=DEPTH+CDEP
```

C  
C  
C  
C  
C  
14

C  
C  
C

C  
C  
C  
22

C  
C  
C  
42



```

D4=DEPTH
TIME=TIME+(-1.000/G)*DLOG((DTAN(P))/(DTAN(P+(F1/2.000))))
T4=TIME
CDEP=-CDEP
X2=((CDEP*G)/C1)+1.000
F2=CARCCS(X2)
Y2=DSIN(F2)
FRAN=(C1*(-Y2))/G
RAN=RAN+FRAN
R4=RAN
IF (RANGE-RAN)51,51,52
  ADD ARC 6
C1=C1+G*CDEF
DEPTH=DEPTH+CDEP
D5=DEPTH
TIME=TIME+(-1.000/G)*DLOG((DTAN(P+F2/2.000))/(DTAN(P)))
T5=TIME
X1=X2
Y1=Y2
G=0.01700
CDEP=(C1*(1.000-X1))/(G*X1)
GRAN=(C1*Y1)/(G*X1)
RAN=GRAN+RAN
R5=RAN
IF (RANGE-RAN)61,61,62
  ADD ARC 7
C1=C1+G*CDEF
DEPTH=DEPTH+CDEP
D6=DEPTH
TIME=TIME+(-1.000/G)*DLOG((DTAN(P))/(DTAN(P+(F2/2.000))))
T6=TIME
CDEP=-CDEP
X2=(CDEP*G)/C1+1.000
F2=-DARCCS(X2)
Y2=DSIN(F2)
HRAN=(C1*(-Y2))/G
RAN=RAN+HRAN
R6=RAN
IF (RANGE-RAN)71,71,92
  ADD ARC 8
C1=C1+G*CDEF
DEPTH=DEPTH+CDEP

```

CC52

CC62

CC92

THE01450  
THE01460  
THE01470  
THE01480  
THE01490  
THE01500  
THE01510  
THE01520  
THE01530  
THE01540  
THE01550  
THE01560  
THE01570  
THE01580  
THE01590  
THE01600  
THE01610  
THE01620  
THE01630  
THE01640  
THE01650  
THE01660  
THE01670  
THE01680  
THE01690  
THE01700  
THE01710  
THE01720  
THE01730  
THE01740  
THE01750  
THE01760  
THE01770  
THE01780  
THE01790  
THE01800  
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THE01840  
THE01850  
THE01860  
THE01870  
THE01880  
THE01890  
THE01900  
THE01910  
THE01920



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THE01930
THE01940
THE01950
THE01960
THE01970
THE01980
THE01990
THE02000
THE02010
THE02020
THE02030
THE02040
THE02050
THE02060
THE02070
THE02080
THE02090
THE02100
THE02110
THE02120
THE02130
THE02140
THE02150
THE02160
THE02170
THE02180
THE02190
THE02200
THE02210
THE02220
THE02230
THE02240
THE02250
THE02260
THE02270
THE02280
THE02290
THE02300
THE02310
THE02320
THE02330
THE02340
THE02350
THE02360
THE02370
THE02380
THE02390
THE02400

D7=DEPTH
TIME=TIME+((-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/(DTAN(P))))
T7=TIME
G=-0.017D0
X1=X2
Y1=Y2
CDEP=(C1*(1.0D0-X1))/(G*X1)
RAN1=C1*Y1/(G*X1)
F1=DARSIN(Y1)
RAN=RAN+RAN1
R7=RAN
IF(RANGE-RAN)101,101,102
    ADC ARC 9
C1=C1+G*CDEF
DEPTH=DEPTH+CDEP
D8=DEPTH
TIME=TIME+((-1.0D0/G)*DLOG((DTAN(P))/(DTAN(P+(F1/2.0D0)))))
T8=TIME
CDEP=-CDEP
X2=((CDEP*G)/C1)+1.0D0
F2=DARCCOS(X2)
Y2=DSIN(F2)
RAN2=(C1*(-Y2))/G
RAN=RAN+RAN2
R8=RAN
IF(RANGE-RAN)111,111,112
    ADC ARC 10
C1=C1+G*CDEF
DEPTH=DEPTH+CDEP
D9=DEPTH
TIME=TIME+((-1.0D0/G)*DLOG((DTAN(P+F2/2.0D0))/(DTAN(P))))
T9=TIME
X1=X2
Y1=Y2
G=0.017D0
CDEP=(C1*(1.0D0-X1))/(G*X1)
RAN3=(C1*Y1)/(G*X1)
RAN=RAN+RAN3
R9=RAN
IF(RANGE-RAN)121,121,122
    ADC ARC 11
C1=C1+G*CDEF
C122

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THE02410  
THE02420  
THE02430  
THE02440  
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THE02460  
THE02470  
THE02480  
THE02490  
THE02500  
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THE02670  
THE02680  
THE02690  
THE02700  
THE02710  
THE02720  
THE02730  
THE02740  
THE02750  
THE02760  
THE02770  
THE02780  
THE02790  
THE02800  
THE02810  
THE02820  
THE02830  
THE02840  
THE02850  
THE02860  
THE02870  
THE02880

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DEPTH=DEPTH+CDEP
D10=DEPTH
TIME=TIME+((-1.0CO/G)*DLOG((DTAN(P))/(DTAN(P+(F2/2.0DO))))))
T10=TIME
CDEP=-CDEP
X2=(CDEP*G)/C1+1.0DO
F2=-DARCS(Y2)
Y2=DSIN(F2)
RAN4=(C1*(-Y2))/G
RAN=RAN+RAN4
R6=RAN
IF (RANGE-RAN)131,131,92

CONTINUE ADCING ARCS IF SUMMED HORIZONTAL RANGE IS LESS THAN
TOTAL RANGE, OTHERWISE EXIT TO 131

CALCULATES TRAVEL TIME OF REFERENCE RAY
Y2=Y1-((RANGE-R1A)*G*X1)/C1
F2=DARCSIN(Y2)
X2=DCOS(F2)
CDEP=C1*(X2-X1)/(G*X1)
DEPTH=DEPTH+CDEP
IF (TIME)7,7,8
TIME=(-1.0DC/G)*DLOG((DTAN(P+(F2/2.0DO)))/(DTAN(P)))+TIME
GO TO 32
TIME=(-1.0DC/G)*DLOG((DTAN(P+(F2/2.0DO)))/(CTAN(P+(A1/2.0DO))))
DEGRE=A1*18C.0DO/PI
N=N+1
IF (A1)72,29,72
K1=K1-1
IF (K1)72,811,72
REFTI=TIME
GO TO 72

SUBROUTINE TO CALCULATE DEPTH AND TIME IF HCRIZONTAL RANGE OF
ARC 1 IS GREATER THAN SOURCE-RECEIVER RANGE
F1=F2
Y2=Y1-((RANGE-R1)*G*X1)/C1
F2=DARCSIN(Y2)
X2=DCOS(F2)
CDEP=C1*(X2-X1)/(G*X1)
TIME=T1+((-1.0DO/G)*DLOG((DTAN(P+(F2/2.0DO)))/(DTAN(P+(F1/2.0DO))))
DEPTH=C1+CDEP
GO TO 32

```



C	21	SUBROUTINE TO CALCULATE DEPTH AND TIME IF ADDITION OF RANGE OF	THE02890
C		ARC 2 CAUSES SUMMED RANGE TO BE GREATER THAN SOURCE-RECEIVER	THE02900
C		RANGE	THE02910
C		Y2=-(RANGE-R11)*G/C1	THE02920
C		F2=DARSIN(Y2)	THE02930
C		X2=DCOS(F2)	THE02940
C		CDEP=C1*(X2-1.0C0)/G	THE02950
C		T1ME=T2+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))	THE02960
C		DEPTH=C2+CDEP	THE02970
C		GO TO 32	THE02980
C		USE IF ARC 3 CAUSES OVERFLOW	THE02990
C		Y2=Y1-((RANGE-R2)*G*X1)/C1	THE03000
C		F2=DARSIN(Y2)	THE03010
C		X2=DCOS(F2)	THE03020
C		CDEP=C1*(X2-X1)/(G*X1)	THE03030
C		T1ME=T3+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0)))	THE03040
C		DEPTH=C3+CDEP	THE03050
C		GO TO 32	THE03060
C		USE IF ARC 4 CAUSES OVERFLOW	THE03070
C		Y2=-(RANGE-R3)*G/C1	THE03080
C		F2=DARSIN(Y2)	THE03090
C		X2=DCOS(F2)	THE03100
C		CDEP=C1*(X2-1.0D0)/G	THE03110
C		T1ME=T4+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P))	THE03120
C		DEPTH=D4+CDEP	THE03130
C		GO TO 32	THE03140
C		USE IF ARC 5 CAUSES OVERFLOW	THE03150
C		F1=F2	THE03160
C		Y2=Y1-((RANGE-R4)*G*X1)/C1	THE03170
C		F2=DARSIN(Y2)	THE03180
C		X2=DCOS(F2)	THE03190
C		CDEP=C1*(X2-X1)/(G*X1)	THE03200
C		T1ME=T5+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0)))	THE03210
C		DEPTH=C5+CDEP	THE03220
C		GO TO 32	THE03230
C		USE IF ARC 6 CAUSES OVERFLOW	THE03240
C		Y2=-(RANGE-R5)*G/C1	THE03250
C		F2=DARSIN(Y2)	THE03260
C		X2=DCOS(F2)	THE03270
C		CDEP=C1*(X2-X1)/(G*X1)	THE03280
C		T1ME=T5+(-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0)))	THE03290
C		DEPTH=C5+CDEP	THE03300
C		GO TO 32	THE03310
C		USE IF ARC 6 CAUSES OVERFLOW	THE03320
C		Y2=-(RANGE-R5)*G/C1	THE03330
C		F2=DARSIN(Y2)	THE03340
C		X2=DCOS(F2)	THE03350
C			THE03360



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CDEP=C1*(X2-1.0D0)/G
TIME=T6+((-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P)))
DEPTH=D6+CDEP
GO TO 32

C
C
C 101
USE IF ARC 7 CAUSES OVERFLOW
Y2=Y1-((RANGE-R6)*G*X1)/C1
F2=DARSIN(Y2)
X2=CCOS(F2)
CDEP=C1*(X2-X1)/(G*X1)
TIME=T7+((-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0))))
DEPTH=D7+CDEP
GO TO 32

C
C
C 111
USE IF ARC 8 CAUSES OVERFLOW
Y2=-(RANGE-F7)*G/C1
F2=DARSIN(Y2)
X2=CCOS(F2)
CDEP=C1*(X2-1.0D0)/G
TIME=T8+((-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P)))
DEPTH=D8+CDEP
GO TO 32

C
C
C 121
USE IF ARC 9 CAUSES OVERFLOW
F1=F2
Y2=Y1-((RANGE-R8)*G*X1)/C1
F2=DARSIN(Y2)
X2=CCOS(F2)
CDEP=C1*(X2-X1)/(G*X1)
TIME=T9+((-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P+(F1/2.0D0))))
DEPTH=D9+CDEP
GO TO 32

C
C
C 131
USE IF ARC 10 CAUSES OVERFLOW
Y2=-(RANGE-F9)*G/C1
F2=DARSIN(Y2)
X2=CCOS(F2)
CDEP=C1*(X2-1.0D0)/G
TIME=T10+((-1.0D0/G)*DLOG((DTAN(P+(F2/2.0D0)))/DTAN(P)))
DEPTH=D10+CDEP
GO TO 32

C
C
C
DEPTH, RELATIVE TIME, AND INITIAL DEPRESSION ANGLE OF RAY ARE
PUT IN A LISTING

```

THE03370  
THE03380  
THE03390  
THE03400  
THE03410  
THE03420  
THE03430  
THE03440  
THE03450  
THE03460  
THE03470  
THE03480  
THE03490  
THE03500  
THE03510  
THE03520  
THE03530  
THE03540  
THE03550  
THE03560  
THE03570  
THE03580  
THE03590  
THE03600  
THE03610  
THE03620  
THE03630  
THE03640  
THE03650  
THE03660  
THE03670  
THE03680  
THE03690  
THE03700  
THE03710  
THE03720  
THE03730  
THE03740  
THE03750  
THE03760  
THE03770  
THE03780  
THE03790  
THE03800  
THE03810  
THE03820  
THE03830  
THE03840



72

```
K=K+1 TIME-REFTI
Y(K)=DEPTH
X(K)=DEPT
WRITE(6,501)X(K),Y(K),DEGRE
WRITE(8,501)X(K),Y(K),DEGRE
FORMAT(F10.4,F14.7,9X,F12.8)
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901

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INCREMENT THE INITIAL DEPRESSION ANGLE BY 0.1 DEGREES AND RESET
INITIAL CONDITIONS. REPEAT THE OPERATION UNTIL MAXIMUM R-R RAY
INITIAL DEPRESSION ANGLE IS REACHED. LAST RAY FROM SOURCE DEPTH
IS AT THIS ANGLE.
```

73

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IF (A1) 58,73,73
A1=A1+(PI*0.1D0)/180.0D0
RANGE=R
DEPTH=C
CI=C
RAN=0.0C0
TIME=0.0C0
Y1=DSIN(A1)
X1=DCOS(A1)
IF (A1+F3) 88,88,57
I=I+1
A1=-F3
Y1=DSIN(A1)
X1=DCOS(A1)
IF (I-2) 48,55,99
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97

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THE FOLLOWING SUBROUTINE IS FOR NEGATIVE (UPWARD) INITIAL
DEPRESSION ANGLES
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96

```
A1=ANGLE
RANGE=R
DEPTH=C
CI=C
A1=A1-(PI*0.1D0)/180.0D0
Y1=DSIN(A1)
X1=DCOS(A1)
IF (A1+(-F3)) 93,53,53
I=I+1
A1=F3
Y1=DSIN(A1)
X1=DCOS(A1)
IF (I-4) 53,95,99
G=-0.017D0
R1A=CI*Y1/(C*X1)
```

93

53

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THE03850
THE03860
THE03870
THE03880
THE03890
THE03900
THE03910
THE03920
THE03930
THE03940
THE03950
THE03960
THE03970
THE03980
THE03990
THE04000
THE04010
THE04020
THE04030
THE04040
THE04050
THE04060
THE04070
THE04080
THE04090
THE04100
THE04110
THE04120
THE04130
THE04140
THE04150
THE04160
THE04170
THE04180
THE04190
THE04200
THE04210
THE04220
THE04230
THE04240
THE04250
THE04260
THE04270
THE04280
THE04290
THE04300
THE04310
THE04320
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82      IF (RANGE-R1)/81,81,82
      CDEP=C1*(1.00-X1)/(G*X1)
      RAN=R1A
      TIME=(-1.00(G/G)*DLOG((DTAN(P))/DTAN(P+(A1/2.000)))
      X1=1.00C
      Y1=0.00C
      DEPTH=CDEP+LEP TH
      C1=C1+G*CDEF
      GO TO 88
81      Y2=Y1-(RANGE*G*X1)/C1
      F2=DARSIN(Y2)
      X2=DCOS(F2)
      TIME=(-1.00(G/G)*DLOG((DTAN(P+(F2/2.000)))/DTAN(P+(A1/2.000)))
      CDEP=C1*(X2-X1)/(G*X1)
      DEPTH=DEPTH+CDEP
      GO TO 32
      K IS THE NUMBER OF RAYS CALCULATED FOR EACH SOURCE DEPTH
      CONTINUE
      WRITE(6,991)DEPTH,K
      WRITE(8,991)DEPTH,K
      FORMAT(1X,NUMBER OF RAYS FOR THE ',F7.2,' METER DEPTH IS',1X,13)
178      WRITE(6,178)
531      WRITE(8,178)
      FORMAT(1X,
      CONTINUE
      INCREMENT THE SOURCE DEPTH 20 METERS UNTIL 600 METERS
      IS EXCEEDED
      IF (DEPTH-600.000)5,9,9
      CONTINUE
      STOP
      END
      DEPTH      RELATIVE TRAVEL TIME      INITIAL ANGLE
1157.4079      0.0012725      0.0000000
1173.6307      0.0022367      0.1000000
1185.6802      0.0028523      0.2000000
1193.6744      0.0031453      0.3000000
1197.7002      0.0031655      0.4000000
1197.8063      0.0031606      0.5000000
1193.9812      0.0022450      0.6000000
1186.1832      0.0013153      0.7000000
1174.3260      0.0000505      0.8000000
1158.2750      -0.0016306      0.9000000
1137.8606      1.0000000

```



NUMBER OF	DEPT	RELATIVE	TRAVEL	TIME	220.00	METER DEPTH	IS	42	THE
112	8688	-0.	037000	-0.	100000000	1.	100000000	THE04810	
11083	8641	-0.	0622330	-0.	120000000	1.	120000000	THE04820	
11048	10123	-0.	0922386	-0.	130000000	1.	130000000	THE04830	
11067	3558	-0.	128722	-0.	140000000	1.	140000000	THE04840	
10962	3846	-0.	169717	-0.	150000000	1.	150000000	THE04850	
913	8616	-0.	212090	-0.	160000000	1.	160000000	THE04860	
862	6216	-0.	255669	-0.	170000000	1.	170000000	THE04870	
805	0437	-0.	299738	-0.	180000000	1.	180000000	THE04880	
753	5937	-0.	343231	-0.	190000000	1.	190000000	THE04890	
638	7140	-0.	386434	-0.	200000000	2.	200000000	THE04900	
580	9217	-0.	427604	-0.	210000000	2.	210000000	THE04910	
522	6477	-0.	467310	-0.	220000000	2.	220000000	THE04920	
465	2753	-0.	504554	-0.	230000000	2.	230000000	THE04930	
405	1722	-0.	538628	-0.	240000000	2.	240000000	THE04940	
354	9462	-0.	569386	-0.	250000000	2.	250000000	THE04950	
305	2015	-0.	596485	-0.	260000000	2.	260000000	THE04960	
254	5738	-0.	620132	-0.	270000000	2.	270000000	THE04970	
165	2658	-0.	639836	-0.	280000000	2.	280000000	THE04980	
133	3509	-0.	655253	-0.	290000000	3.	290000000	THE04990	
104	2518	-0.	667447	-0.	300000000	3.	300000000	THE05000	
81	3323	-0.	675970	-0.	310000000	3.	310000000	THE05010	
65	3071	-0.	680558	-0.	320000000	3.	320000000	THE05020	
57	8515	-0.	683573	-0.	330000000	3.	330000000	THE05030	
47	8950	-0.	684673	-0.	340000000	3.	340000000	THE05040	
38	7971	-0.	684959	-0.	350000000	3.	350000000	THE05050	
18	3067	-0.	685065	-0.	360000000	3.	360000000	THE05060	
16	4389	-0.	686560	-0.	370000000	3.	370000000	THE05070	
15	6786	-0.	690506	-0.	380000000	3.	380000000	THE05080	
14	2553	-0.	699827	-0.	390000000	3.	390000000	THE05090	
13	6786	-0.	715197	-0.	400000000	4.	400000000	THE05100	
12	3486	-0.	725201	-0.	404685864	4.	404685864	THE05110	
11	3486	-0.	725201	-0.	404685864	4.	404685864	THE05120	
10	3486	-0.	725201	-0.	404685864	4.	404685864	THE05130	
9	3486	-0.	725201	-0.	404685864	4.	404685864	THE05140	
8	3486	-0.	725201	-0.	404685864	4.	404685864	THE05150	
7	3486	-0.	725201	-0.	404685864	4.	404685864	THE05160	
6	3486	-0.	725201	-0.	404685864	4.	404685864	THE05170	
5	3486	-0.	725201	-0.	404685864	4.	404685864	THE05180	
4	3486	-0.	725201	-0.	404685864	4.	404685864	THE05190	
3	3486	-0.	725201	-0.	404685864	4.	404685864	THE05200	
2	3486	-0.	725201	-0.	404685864	4.	404685864	THE05210	
1	3486	-0.	725201	-0.	404685864	4.	404685864	THE05220	
0	3486	-0.	725201	-0.	404685864	4.	404685864	THE05230	
0	3486	-0.	725201	-0.	404685864	4.	404685864	THE05240	
0	3486	-0.	725201	-0.	404685864	4.	404685864	THE05250	
0	3486	-0.	725201	-0.	404685864	4.	404685864	THE05260	
0	3486	-0.	725201	-0.	404685864	4.	404685864	THE05270	
0	3486	-0.	725201	-0.	404685864	4.	404685864	THE05280	



1305.5720  
 11273.11330  
 112336.54325  
 111332.25275  
 110774.42654  
 10035.38578  
 8682.19706  
 788.7754  
 714.7250  
 641.9417  
 565.0637  
 500.7447  
 432.6572  
 368.9088  
 308.4811  
 252.6505  
 202.9175  
 158.9356  
 120.9398  
 90.1725  
 65.3833  
 53.3308  
 36.7798  
 19.5029  
 7.2802  
 1153.8985  
 167.9190  
 NUMBER CF

0.0111410  
 0.0086388  
 0.0055768  
 0.00199620  
 -0.00233034  
 -0.00273848  
 -0.00197747  
 -0.00260831  
 -0.00378035  
 -0.00431235  
 -0.00479999  
 -0.00523572  
 -0.00566407  
 -0.00596466  
 -0.00627361  
 -0.00664672  
 -0.00677407  
 -0.00685519  
 -0.00690749  
 -0.00692636  
 -0.00693339  
 -0.00693250  
 -0.00694228  
 -0.00697576  
 -0.00706456  
 -0.00721825  
 -0.00727444  
 RAYS FCR THE

240.00 METER DEPTH IS 44

THE05290  
 THE05300  
 THE05310  
 THE05320  
 THE05330  
 THE05340  
 THE05350  
 THE05360  
 THE05370  
 THE05380  
 THE05390  
 THE05400  
 THE05410  
 THE05420  
 THE05430  
 THE05440  
 THE05450  
 THE05460  
 THE05470  
 THE05480  
 THE05490  
 THE05500  
 THE05510  
 THE05520  
 THE05530  
 THE05540  
 THE05550  
 THE05560  
 THE05570  
 THE05580  
 THE05590  
 THE05600



APPENDIX B  
INTERPOLATION OF RELATIVE TRAVEL TIME CALCULATIONS

This program does an interpolation of the output data generated in Appendix A. The relative travel times are interpolated for the receiving hydrophone depths.









```

C      EVALUATES THE SLICE AT THESE POINTS
C      U(1)=100.0
C      U(2)=200.0
C      U(3)=300.0
C      U(4)=400.0
C      U(5)=500.0
C      CALL ICSEVU(X,Y,NX,C,K1,U,S,5,IER)
C      OUTPUT ARE TRAVEL TIMES PRINTED BY ROW. THE FIRST ROW IS FOR
C      THE SHALLOWEST SOURCE DEPTH AND LAST ROW IS FOR THE DEEPEST
C      SOURCE DEPTH. THE FIRST COLUMN IS FOR THE SHALLOWEST RECEIVING
C      HYDROPHONE AND THE FINAL COLUMN IS FOR THE DEEPEST HYDROPHONE.
C      WRITE(6,101)(S(I),I=1,5)
C      WRITE(7,101)(S(I),I=1,5)
C      FORMAT(5F14.7)
C      101
C      PROGRAM TERMINATES WHEN NUMBER OF RAYS FOR A SOURCE DEPTH EQUALS
C      THE NUMBER OF RAYS FOR THE DEEPEST SOURCE DEPTH
C      IF(K(IJ)-68)2,21,21
C      21
C      CONTINUE
C      STOP
C      END
$ENTRY
-0.0654165-0.0658337-0.0621516-0.0574149-0.0518334
-0.0703212-0.0665503-0.0628172-0.0580503-0.0524011
-0.0713123-0.0673456-0.0635689-0.0587241-0.0530590
-0.0723207-0.0681941-0.0643641-0.0595105-0.0537945
-0.0745428-0.0695940-0.0652127-0.0603237-0.0546137
-0.0756658-0.0709912-0.0670775-0.0621397-0.0563615
-0.0747207-0.0720147-0.0680565-0.0631501-0.0572667
-0.0758055-0.0731056-0.0691335-0.0641502-0.0583075
-0.0769575-0.0742301-0.0701951-0.0652131-0.0593780
-0.0781758-0.0753760-0.0713190-0.0663189-0.0604718
-0.0793888-0.0765576-0.0724953-0.0674813-0.0616073
-0.0806178-0.0777900-0.0737091-0.0686366-0.0627376
-0.0819152-0.0790446-0.0749412-0.0698603-0.0639641
-0.0832825-0.0803677-0.0762265-0.0711268-0.0652111
-0.0846354-0.0817073-0.0775561-0.0724360-0.0664785
-0.0860246-0.0830759-0.0788332-0.0737404-0.0677993
-0.0874630-0.0844590-0.0802819-0.0751390-0.0691366
-0.0888928-0.0859060-0.0816601-0.0764943-0.0705184
-0.0902551-0.0873991-0.0830757-0.0789512-0.0719424

```



APPENDIX C  
RESULTING BEAM PATTERN FOR CALCULATED WEIGHTS

This program uses the output of Appendix B to calculate the 'A' matrix. From this, the amplitude and phase weights are determined by the linear minimum variance estimation technique. These weights are applied to the array and the beam pattern is obtained.













```

999      WRITE(6,999)
        FORMAT(IX,ANS MATRIX (PHASE AND AMPLITUDE WEIGHTS) FOLLOWS.)
158      WRITE(6,158)(ANS(I8,1),I8=1,10)
        FORMAT(F14.7)
C
C      APPLYING THESE PHASE AND AMPLITUDE WEIGHTS, THE RESULTING BEAM
C      PATTERN IS CALCULATED (REAL AND IMAGINARY PARTS SEPARATE)
C
760      CALL VMLFF(A,ANS, 40,10,1,40,10,Y,40,IER2)
C
C      THE REAL AND IMAGINARY PARTS OF THE RESULTING BEAM PATTERN
C      ARE COMBINED TO GIVE ABSOLUTE VALUE.
C
987      I20=I20+1
        XX(I20,1)=SQRT(Y(I20,1)*Y(I20,1)+Y(I20+20,1)*Y(I20+20,1))
        IF(I20-20)987,654,654
        I19=I19+1
        X1(I19,1)=2(0.0+I19*20.0
        IF(I19-20)654,655,655
C
C      PRINT SOURCE DEPTH AND ABSOLUTE VALUE OF BEAM PATTERN AT
C      THAT DEPTH
C
655      WRITE(6,988)(X1(I,1),XX(I,1),I=1,20)
        WRITE(9,988)(X1(I,1),XX(I,1),I=1,20)
988      FORMAT(F8.2,' METER DEPTH HAS STRENGTH CF ',F14.7)
172      CONTINUE
        STOP
$ENTRY
ANS MATRIX (PHASE AND AMPLITUDE WEIGHTS) FOLLOWS
-0.0050524
2.275893C
0.5812412
-3.606538C
-4.330297C
0.2429787
-1.373913C
-3.074544C
2.144663C
C.7122661
MA900570
MA900580
MA900590
MA901000
MA901010
MA901020
MA901030
MA901040
MA901050
MA901060
MA901070
MA901080
MA901090
MA901100
MA901110
MA901120
MA901130
MA901140
MA901150
MA901160
MA901170
MA901180
MA901190
MA901200
MA901210
MA901220
MA901230
MA901240
MA901250
MA901260
MA901270
MA901280
MA901290
MA901300
MA901310
MA901320
MA901330
MA901340
MA901350
MA901360
MA901370
MA901380
MA901390
MA901400
MA901410
MA901420
MA901430
MA901440

```



22	C.	00	ME	TE	DE	HAS	STRENGTH	0.	743061
24	C.	00	ME	TE	DE	HAS	STRENGTH	0.	2850570
26	C.	00	ME	TE	DE	HAS	STRENGTH	0.	2128562
28	C.	00	ME	TE	DE	HAS	STRENGTH	0.	2804181
30	C.	00	ME	TE	DE	HAS	STRENGTH	0.	0716652
32	C.	00	ME	TE	DE	HAS	STRENGTH	0.	0349326
34	C.	00	ME	TE	DE	HAS	STRENGTH	0.	1124682
36	C.	00	ME	TE	DE	HAS	STRENGTH	0.	0530375
38	C.	00	ME	TE	DE	HAS	STRENGTH	0.	0693972
40	C.	00	ME	TE	DE	HAS	STRENGTH	0.	1343069
42	C.	00	ME	TE	DE	HAS	STRENGTH	0.	1722233
44	C.	00	ME	TE	DE	HAS	STRENGTH	0.	1437957
46	C.	00	ME	TE	DE	HAS	STRENGTH	0.	0185341
48	C.	00	ME	TE	DE	HAS	STRENGTH	0.	0239681
50	C.	00	ME	TE	DE	HAS	STRENGTH	0.	0197991
52	C.	00	ME	TE	DE	HAS	STRENGTH	0.	0891870
54	C.	00	ME	TE	DE	HAS	STRENGTH	0.	0190372
56	C.	00	ME	TE	DE	HAS	STRENGTH	0.	1875219
58	C.	00	ME	TE	DE	HAS	STRENGTH	0.	0271981
60	C.	00	ME	TE	DE	HAS	STRENGTH	0.	0740827
MA901450									
MA901460									
MA901470									
MA901480									
MA901490									
MA501500									
MA901510									
MA901520									
MA901530									
MA901540									
MA901550									
MA901560									
MA901570									
MA901580									
MA901590									
MA901600									
MA901610									
MA901620									
MA901630									
MA901640									
MA901650									



## LIST OF REFERENCES

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Thesis

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c.1          Amplitude shading  
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